Model calibration for prediction of pillar failure – CR example

By Terry Wiles
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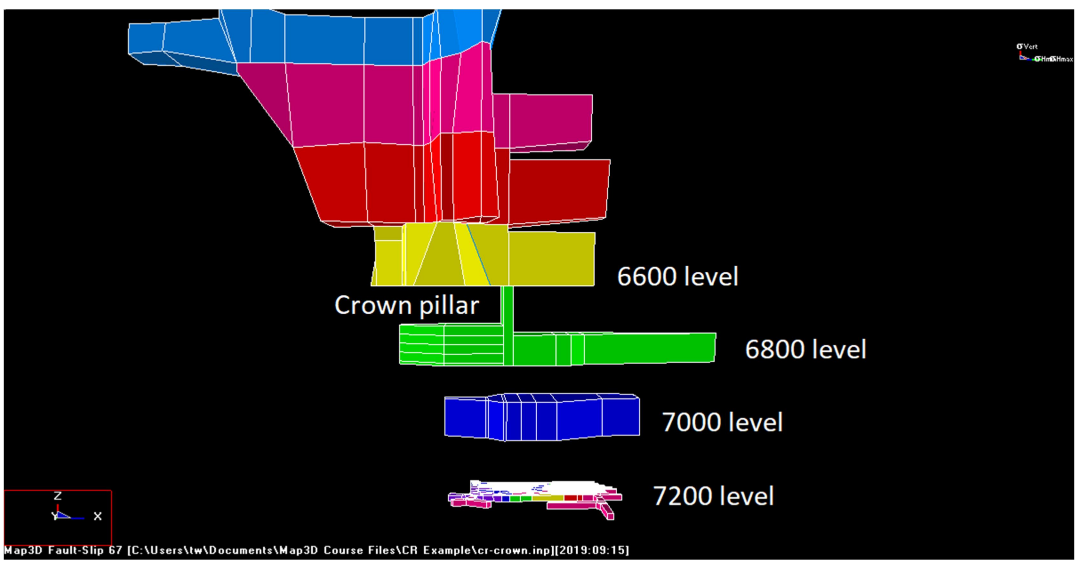
Files required:

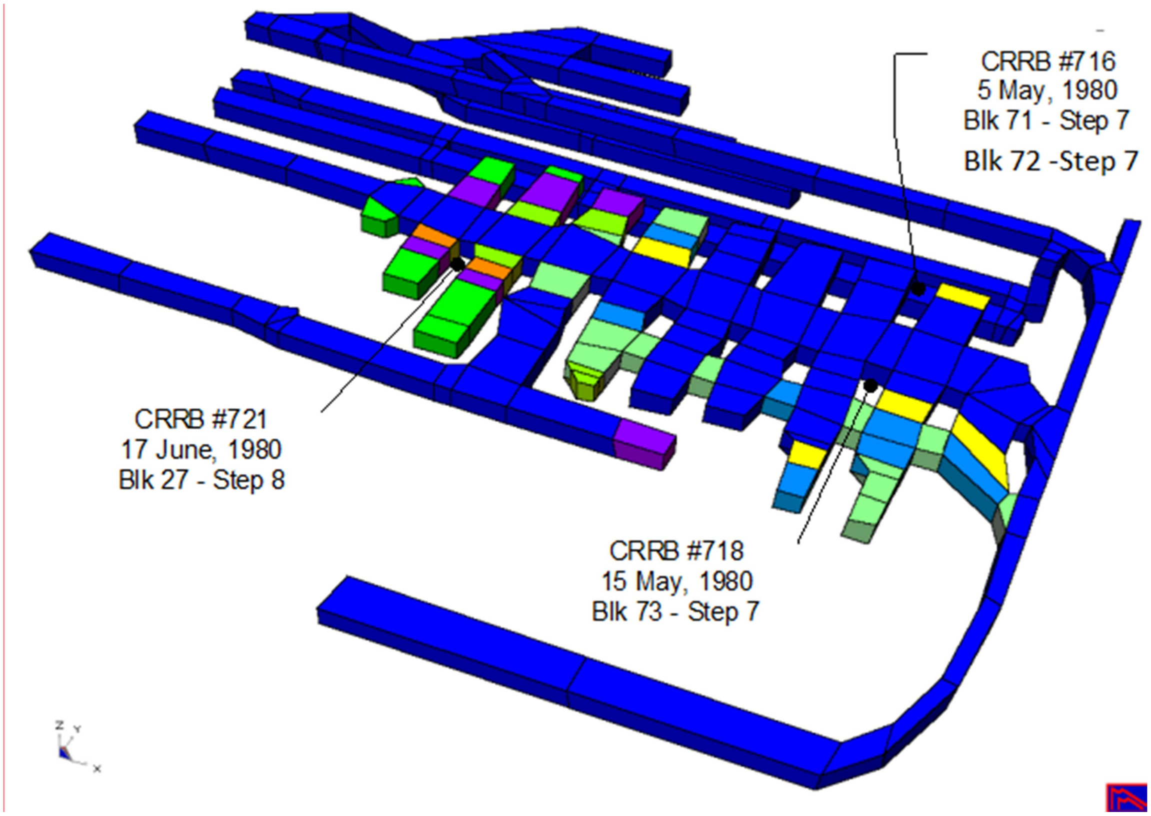
- 6800ERR2.001 to 6800ERR2.008
- 7000ERR.001 to 7000ERR.007
- 7200ERR.001 to 7200ERR.006
- cr-crown.001 to cr-crown.009
- CR example.xlsx

In this example, multiple pillar failures occurred during the silling out stage on 3 different levels (6800, 7000 and 7200). These are used to calibrate a model to make predictions of expected behaviour for mining a subsequent crown pillar at 6800 level. The mining of the sill pillars is considered in multiple steps corresponding to each observed pillar failure. The base depth for levels 6800, 7000 and 7200 (depth in feet) are respectively 2073m, 2134m and 2195m below ground surface.



6800 level - Calibration

Silling out of this level was modelled according to the designed shape "Map3D > File > Results View > 6800ERR2.001". Previous mining is shown in blue. Subsequent mining steps during the period of observed pillar failures are shown in various colours. Note that all of these pillars failed by violent bursting.

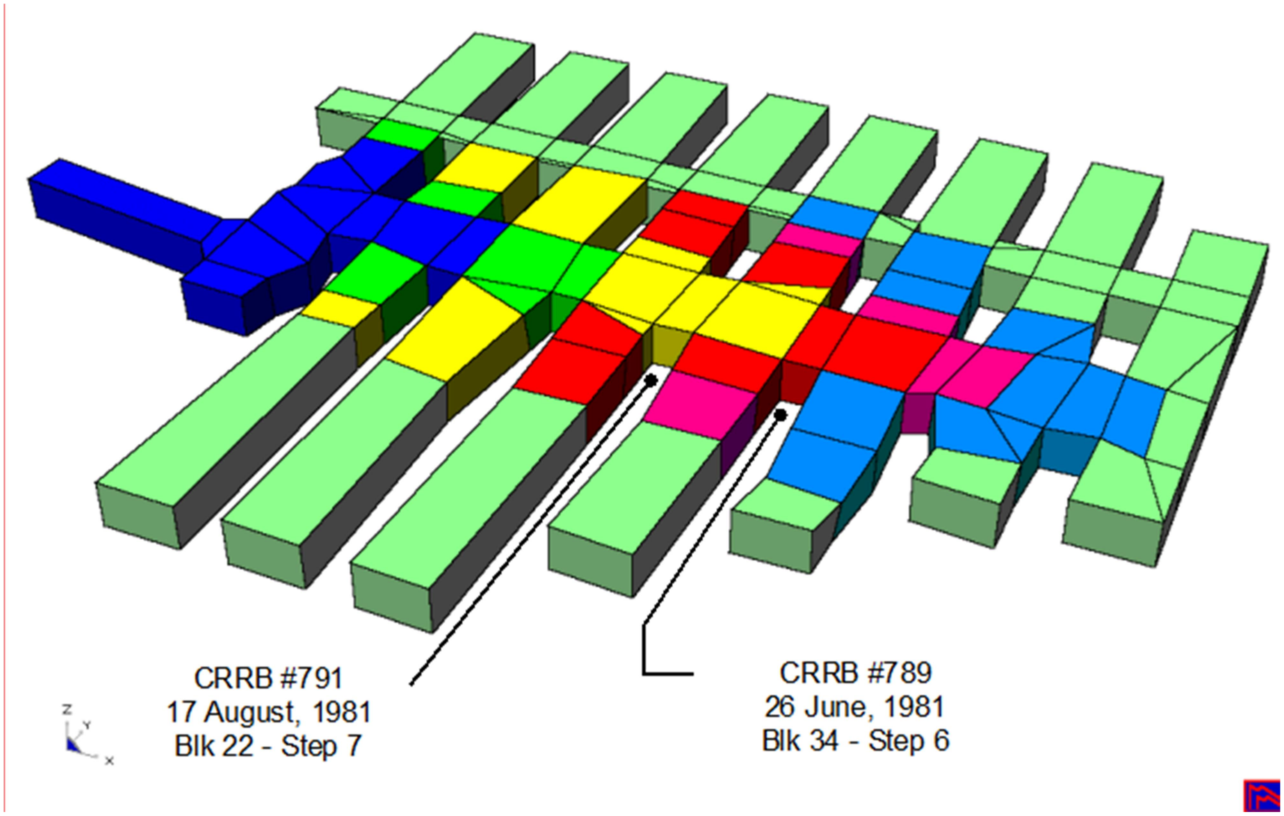


Stresses are calculated at the centre of each of the observed pillar failures at the point in the sequence when the failure occurred.

s3	s1	s2	e3	Level	Step	Block	Grid
18.67831	171.3287811	83.30758667	-651.895	6800	7	71	33
24.22271	221.9219818	56.44063187	-657.506	6800	7	72	34
33.99432	242.3837738	55.60523224	-586.999	6800	7	73	35
21.7537	220.9651337	41.36268997	-635.192	6800	8	7	7

7000 level - Calibration

Silling out of this level was modelled according to the designed shape “Map3D > File > Results View > 7000ERR.001”. Previous mining is shown in blue. Subsequent mining steps during the period of observed pillar failures are shown in various colours. Note that all of these pillars failed by violent bursting.

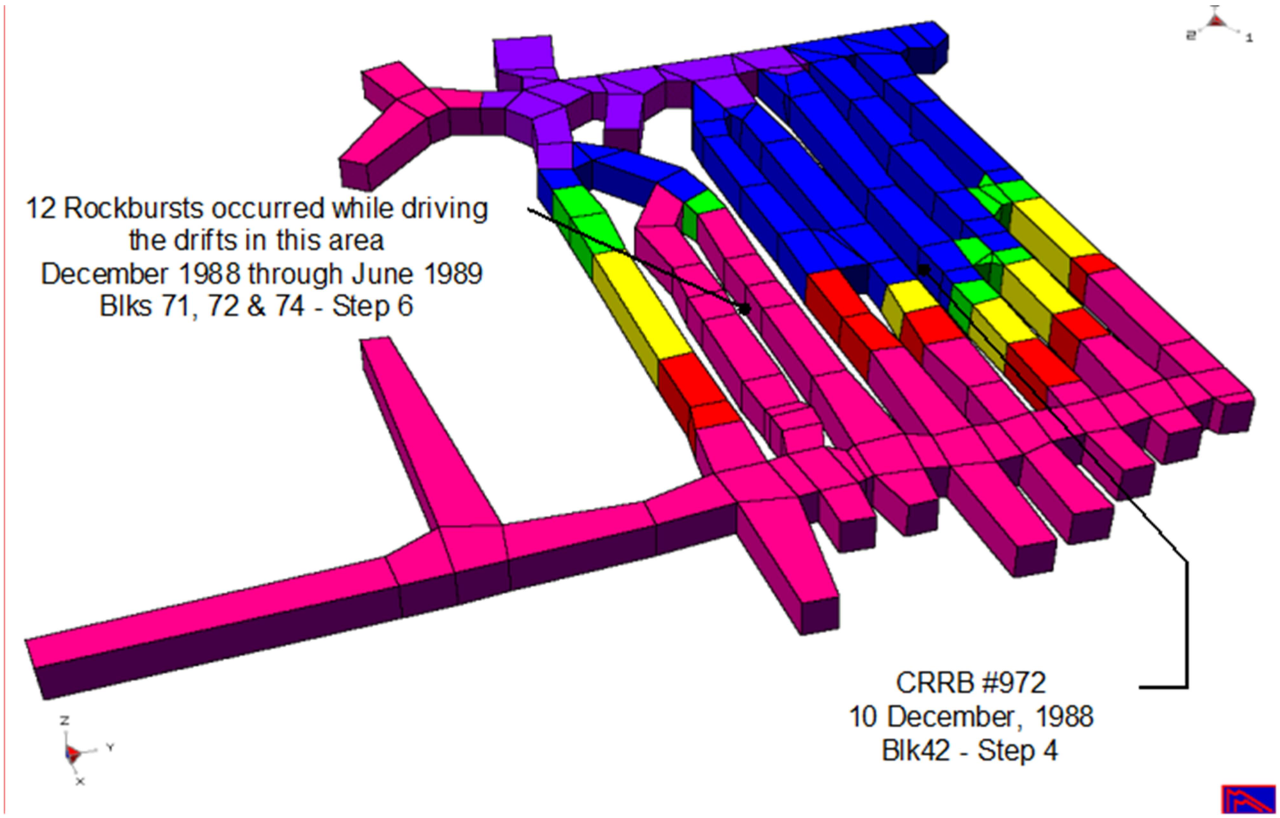


Stresses are calculated at the centre of each of the observed pillar failures at the point in the sequence when the failure occurred.

s3	s1	s2	e3	Level	Step	Block	Grid
19.71889	209.5440674	58.09474564	-683.925	7000	6	34	19
25.1739	227.3088531	66.01667023	-697.934	7000	7	22	7

7200 level - Calibration

Silling out of this level was modelled according to the designed shape "Map3D > File > Results View > 7200ERR.001". Previous mining is shown in blue. Subsequent mining steps during the period of observed pillar failures are shown in various colours. Note that all of these pillars failed by violent bursting.



Stresses are calculated at the centre of each of the observed pillar failures at the point in the sequence when the failure occurred.

s3	s1	s2	e3	Level	Step	Block	Grid
2.706708	133.2966156	94.73170471	-786.962	7200	4	42	12
10.82374	146.5533447	55.03289032	-573.519	7200	5	33	35
0.503497	146.0597992	96.5087204	-871.574	7200	6	72	39
9.089082	149.7138367	97.54698944	-764.147	7200	6	74	40

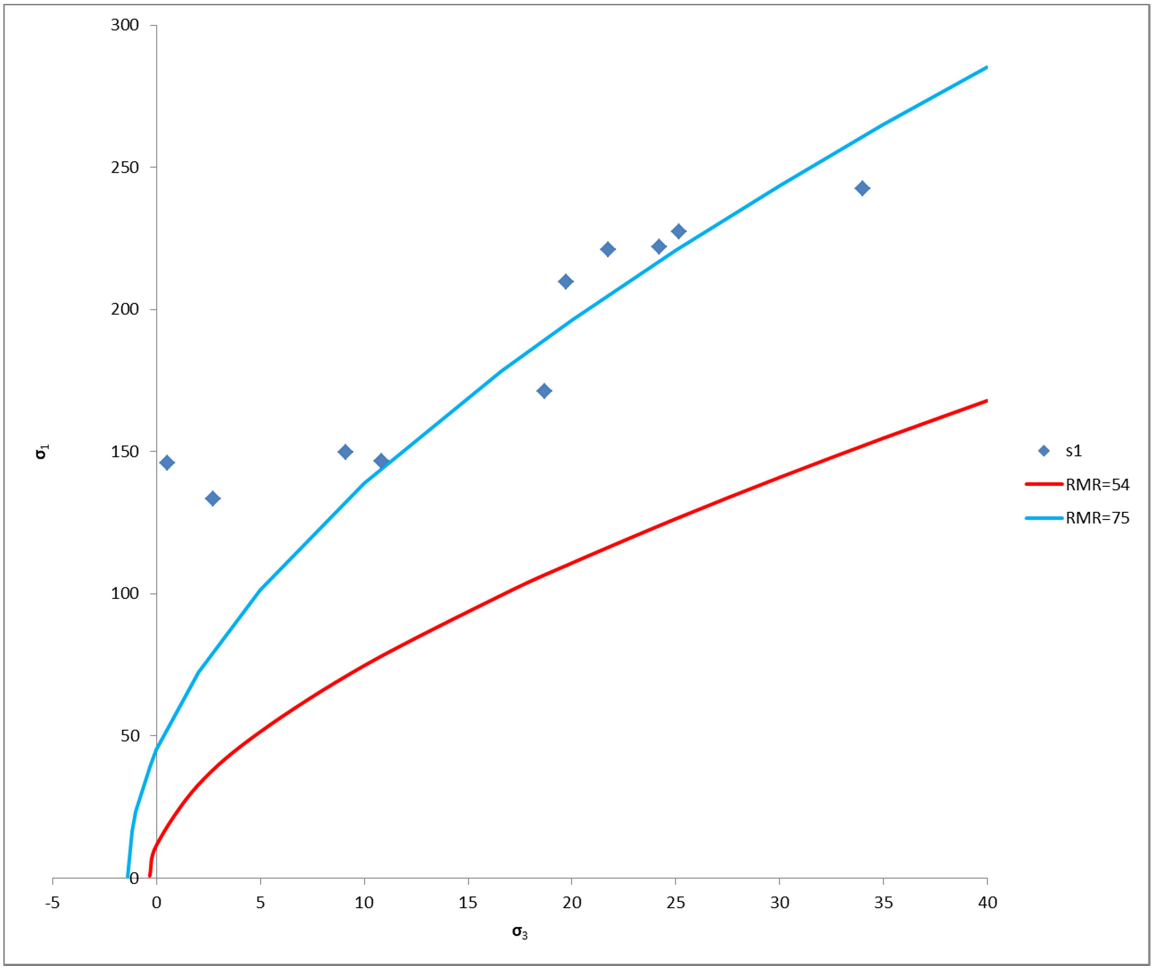
Rock Mass Failure

The mine had developed an Hoek-Brown failure criterion. There was considerable uncertainty regarding both the rock-mass rating and core strength. RMR varies between 54 and 75, depending on where this was measured σ_c^{50} varied between 152-182 MPa for UCS tests conducted wet, and 225-229 MPa for UCS test conducted oven dried. The m_i value was only measured for the oven dried samples and found to be in the range from 13.8 to 19.5.

Based on the ISRM "Suggested Method for Determining the Uniaxial Compressive Strength", and the fact that the field conditions in the mine are wet, it is felt that the oven dried σ_c^{50} is not representative of environment. Based on these numbers, the best estimate for the field scale values are as follows.

RMR	σ_c^{50}	m_i	m	s
54	152 MPa	13.8	2.67	0.00603
75	182 MPa	19.5	7.98	0.062

These are compared to the observed sill pillar failures (blue diamonds) below. It can be observed that the lower Hoek-Brown estimate (*RMR* = 54) severely underestimates the observed pillar failure stress. The upper Hoek-Brown estimate (*RMR* = 75) underestimates the observed pillar failure stress for lower confinements, but does line up pretty well at higher confinements.



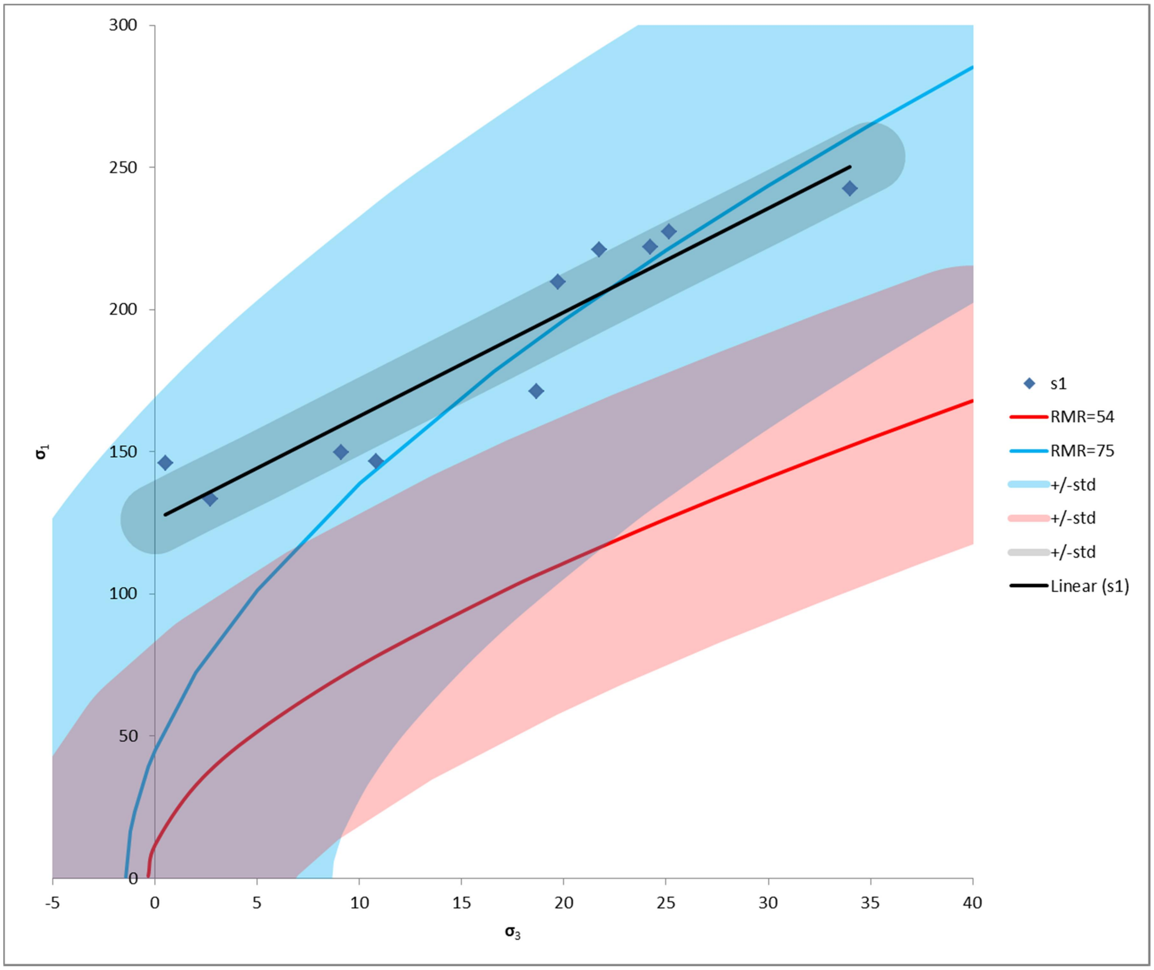
Prediction Uncertainty

There is uncertainty in any model prediction due to uncertainty of the input parameters including the geometry ($C_{Geometry}$), geology ($C_{Geology}$), pre-mining stress state ($C_{\sigma f}$), 50mm core strength ($C_{\sigma c50}$), *RMR* (C_{RMR}), Hoek-Brown strength reduction formula ($C_{Hoek\text{-}Brown}$), numerical errors ($C_{Numerical}$) and local yielding not accommodated by the elastic modelling ($C_{Elasticity}$). Here, uncertainty is expressed as coefficient of variation defined as standard deviation divided by the mean. These uncertainties can be combined using the point estimate method, however by assuming each contribution to uncertainty is normally distributed and uncorrelated, these can also be added superimposed to obtain a combined uncertainty ($C_{Combined}$) using the following relation

$$C^2_{Combined} = C^2_{Geometry} + C^2_{Geology} + C^2_{\sigma f} + C^2_{\sigma c50} + C^2_{RMR} + C^2_{Hoek\text{-}Brown} + C^2_{Numerical} + C^2_{Elasticity}$$

The uncertainty arises mostly from the variability in the measurements of pre-mining stress state $C_{\sigma f}$ (±20-30%), strength parameter σ_{c50} (±20-30%), *RMR* (±?) and the Hoek-Brown strength reduction formula (±?). These later two items are likely considerably larger than ±20-30%. The remaining parameters are thought to be smaller and not significant contributors here (although some would argue that $C^2_{Elasticity}$ is dominatingly large in value). Setting all of $C_{\sigma f}$, $C_{\sigma c50}$, C_{RMR} and $C_{Hoek\text{-}Brown}$ to either ±20%, then ±30%, $C^2_{Combined}$ can be calculated to be on the order of ±40% to ±60%. This estimate can be considered to be on the conservative side and is likely of larger value.

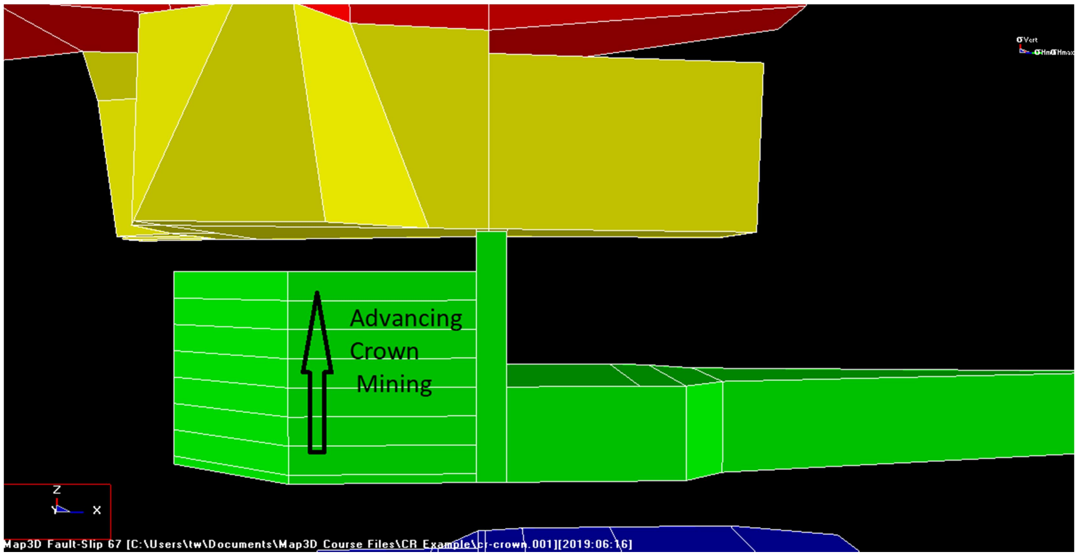
Below, the expected standard deviation is shown for the Hoek-Brown criterion using ±50% for $C_{Combined}$ and noting that by definition coefficient of variation $C = std/\sigma_{mean}$. Also shown is the standard deviation (±1 std) for the best fit line though the observed pillar failures (blue diamonds). In all cases σ_{mean}, has been set equal to the σ_1 value for each respective criterion at the mean of the σ_3 values for the observed pillar failures (σ_{3Mean} = 16.7 MPa).



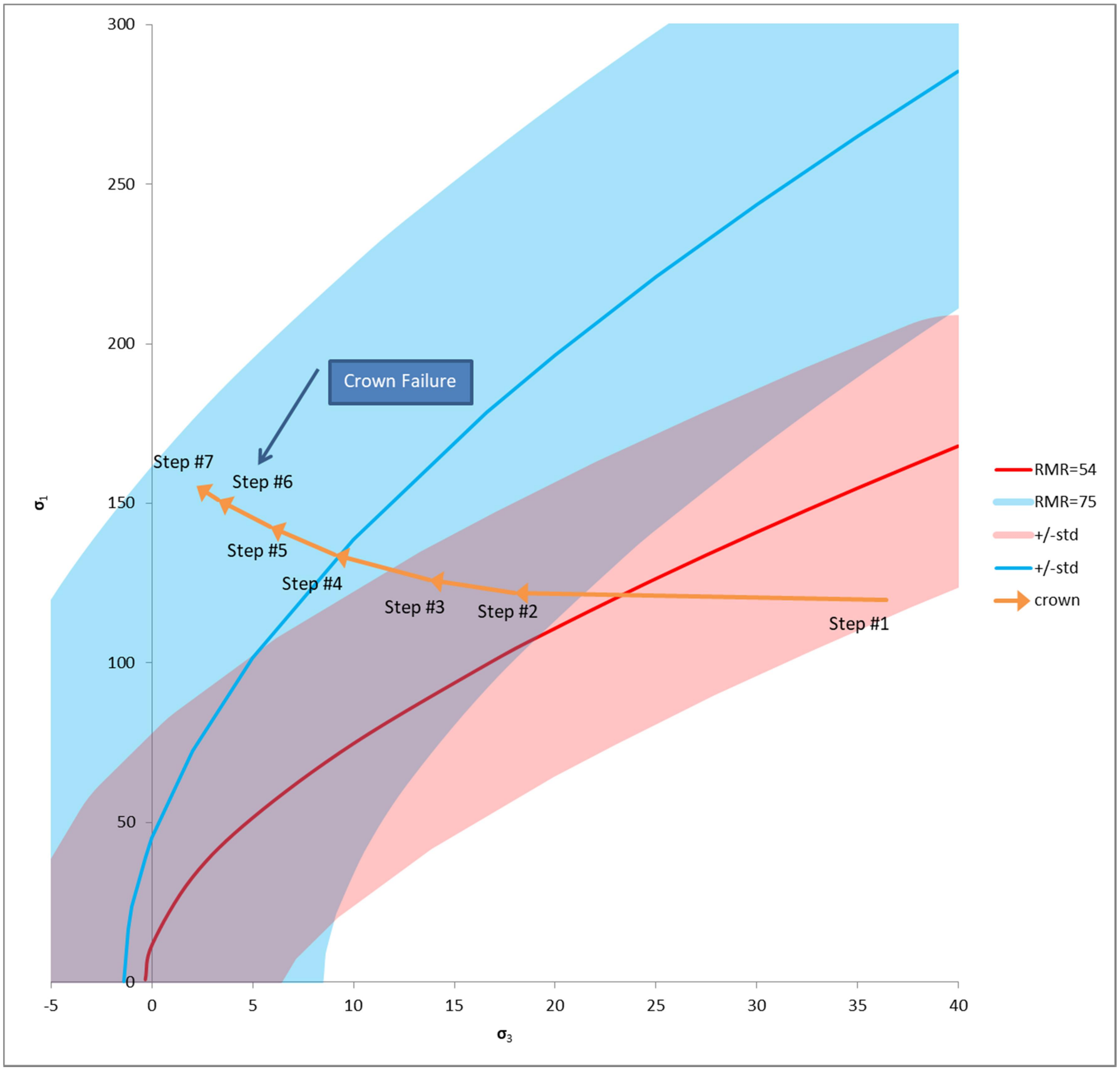
Note that the ±1 standard deviation zone shown corresponds to a confidence interval between 25% and 75% probability. For design purposes, it is more common to use ±2 standard deviation zone which corresponds to a confidence interval between 5% and 95% probability.

Crown Pillar Prediction

Several years after the silling out, mining of the crown pillar was advancing as shown below “Map3D > File > Results View > cr-crown.001”. This is modelled in 2-cut, or 1 year intervals (each cut takes approximately 6 months to complete).



Stresses near the centre of the crown pillar are taken at each step as shown below in orange.



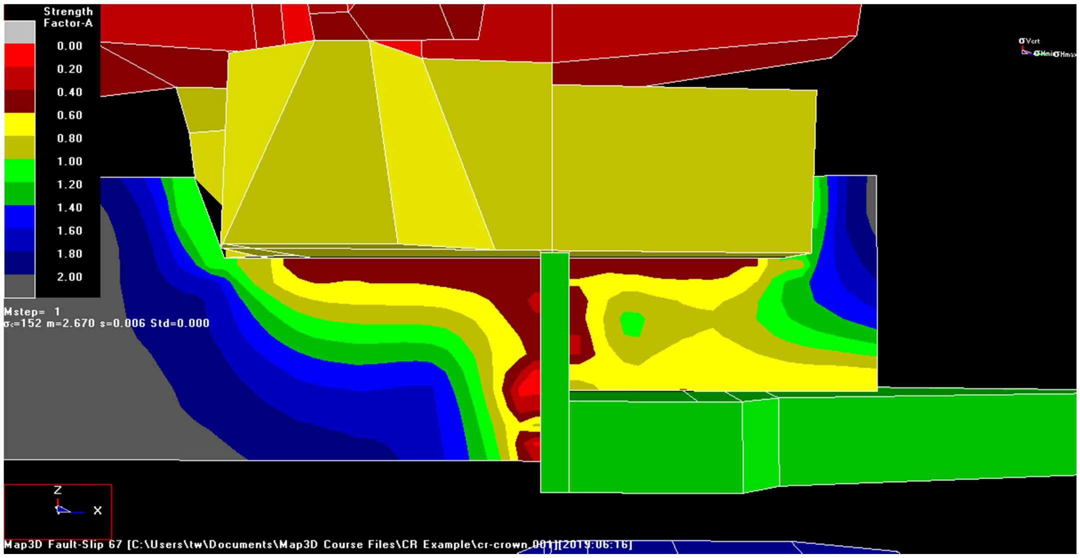
The Hoek-Brown low estimate gives an expected failure of the crown pillar sometime between step #1 and step #2 with a 1 standard deviation uncertainty (confidence interval between 25% and 75% probability) of about 1 to 2 steps (years). A 2 standard deviation uncertainty (confidence interval between 5% and 95% probability) would give an uncertainty that encompassed the entire pillar width.

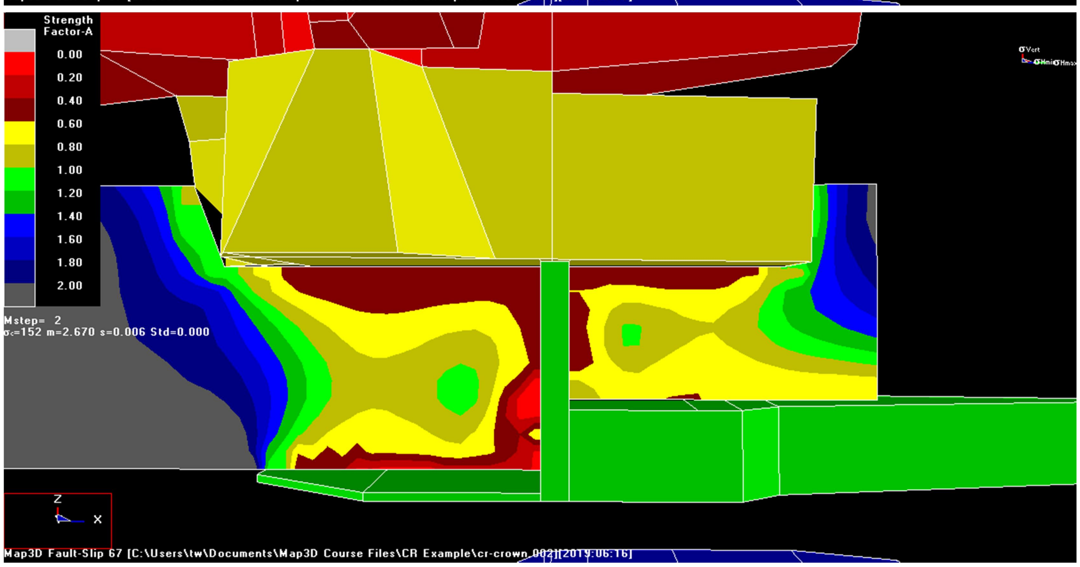
The Hoek-Brown high estimate gives an expected failure of the crown pillar at step #4 with a 1 standard deviation uncertainty (confidence interval between 25% and 75% probability) of about 3 steps (years), nearly the entire pillar width.

Note that this crown pillar did fail by bursting at 36m width, step #6. The Hoek-Brown estimate has failed in two ways here:

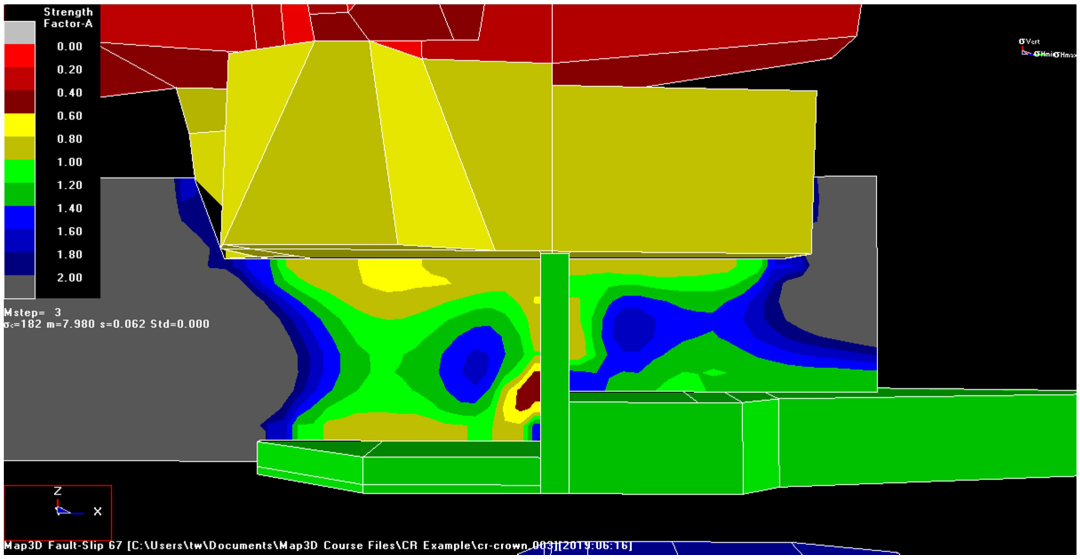
1) The stress at the time of failure is severely underestimated;
2) The uncertainty of stress the time of failure is very large.

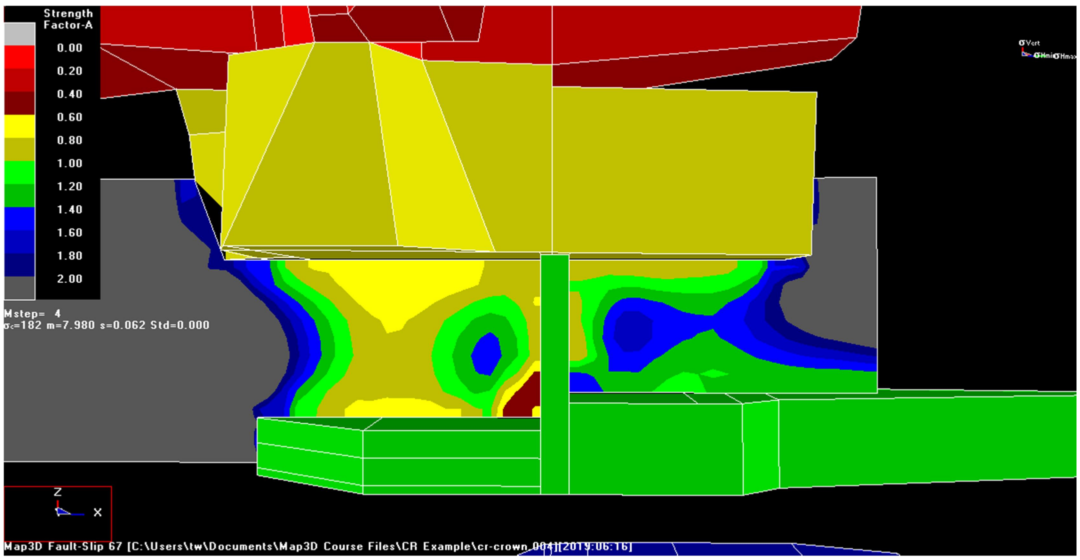
Below are shown the strength factor for the low estimate defined as $\{\sigma_3 + (m \times \sigma_{c50} \times \sigma_3 + s \times \sigma_{c50}{}^2)^{½}\}/\sigma_1$ at Step #1 and Step #2.



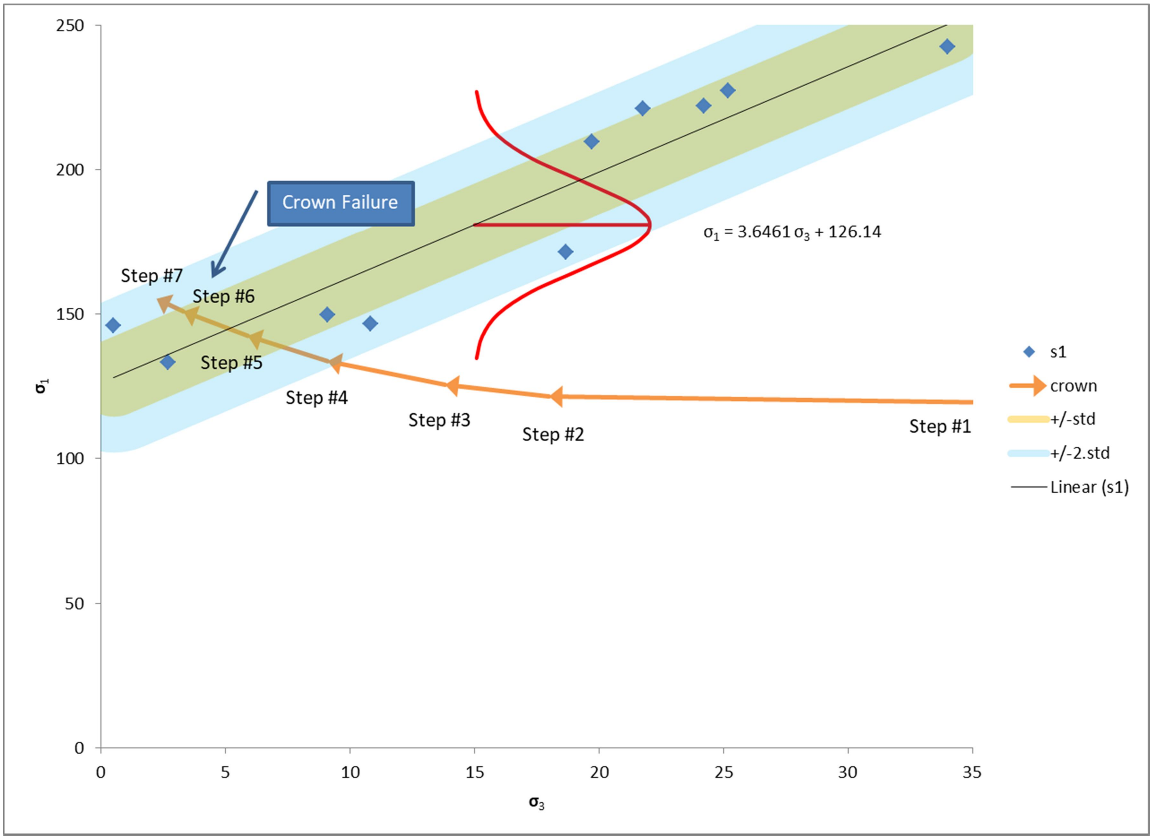


Below are shown the strength factor for the high estimate defined as $\{\sigma_3 + (m \times \sigma_{c50} \times \sigma_3 + s \times \sigma_{c50}{}^2)^{½}\}/\sigma_1$ at Step #3 and Step #4.





Now consider a failure prediction made from the calibration for the silling out on 6800, 7000 and 7200 levels (blue diamonds below).



The best fit line through the calibration points is shown in black. This line corresponds to a Mohr-Coulomb criterion with an intercept *UCS*=126 and slope *q*=3.65. The fiction angle can be calculated as 35° from the relation $q = tan^2(45 + \varphi/2)$.

The standard deviation of the calibration points around this bet fit line can be calculated using the Excel STEYX function and found to equal to ±15.4 MPa. Here, the mean stress can be determined from the average of the σ_1 values (σ_{1Mean} = 187 MPa) for all calibration points. This is also equal to the σ_1 value on the best fit line taken at the mean of the σ_3 values (σ_{3Mean} = 16.7 MPa). Dividing the standard deviation by the mean gives a coefficient of variation of ±8%.

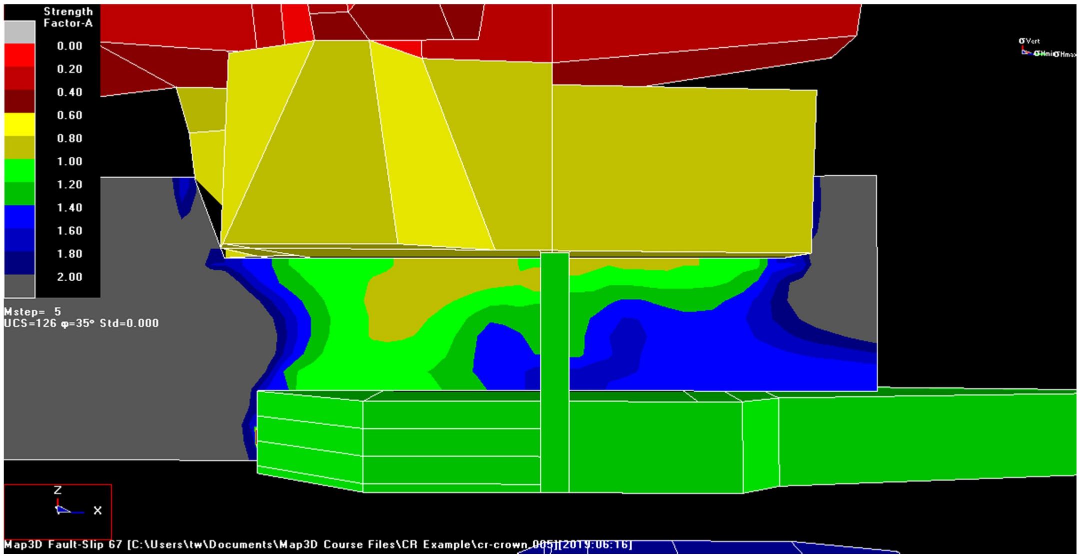
This gives an expected failure of the crown pillar between step #5 and step #6 with a 1 standard deviation uncertainty (confidence interval between 25% and 75% probability) of about 1 step (year). A 2 standard deviation uncertainty (confidence interval between 5% and 95% probability) would give an uncertainty that encompassed 1.5 to 2 steps (years).

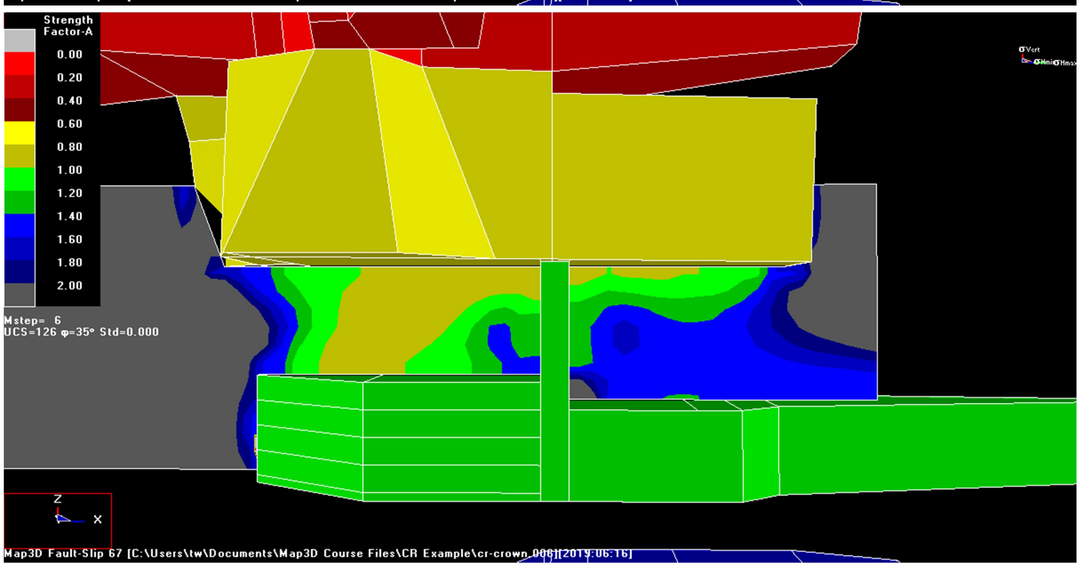
This estimate has improved in two ways here:

1) The stress at the time of failure is quite accurate;
2) The uncertainty of stress the time of failure is relatively small.

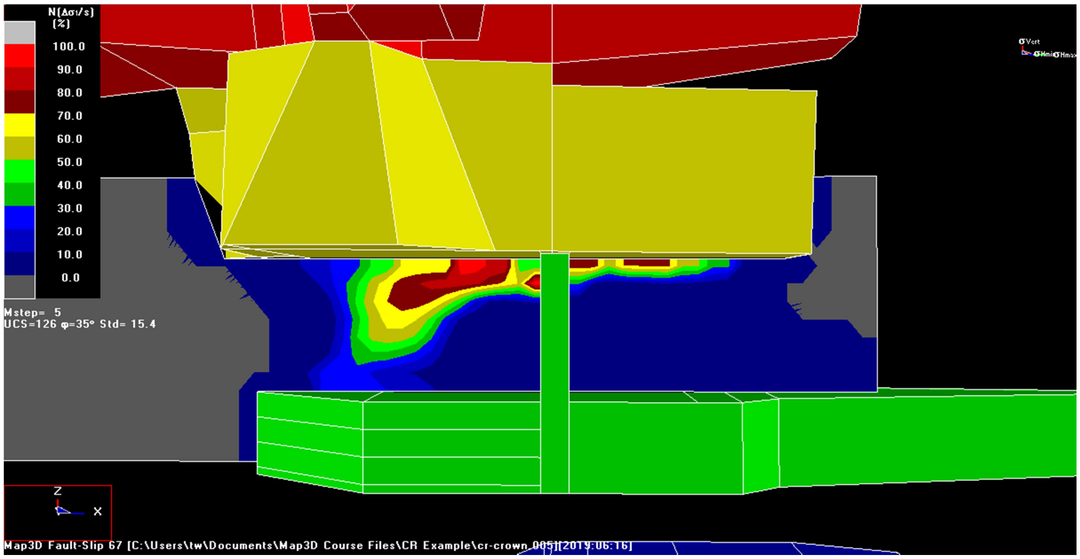
It would appear that this calibration provides a useful predictor of failure.

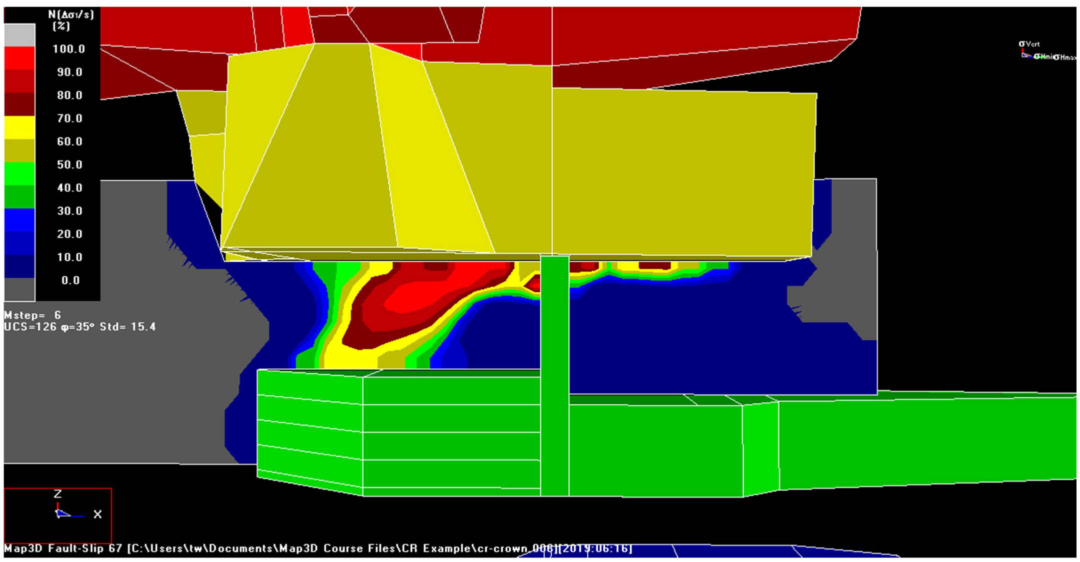
Below are shown the strength factor defined as ($UCS + q \times \sigma_3$) / σ_1 at Step #5 and Step #6.





Below are shown the probability of failure defined as $N(\Delta\sigma_1 / std)$ where the function N represents the normal distribution and the symbol *std* represents the standard deviation and $\Delta\sigma_1 = \sigma_1 - (UCS + q \times \sigma_3)$.

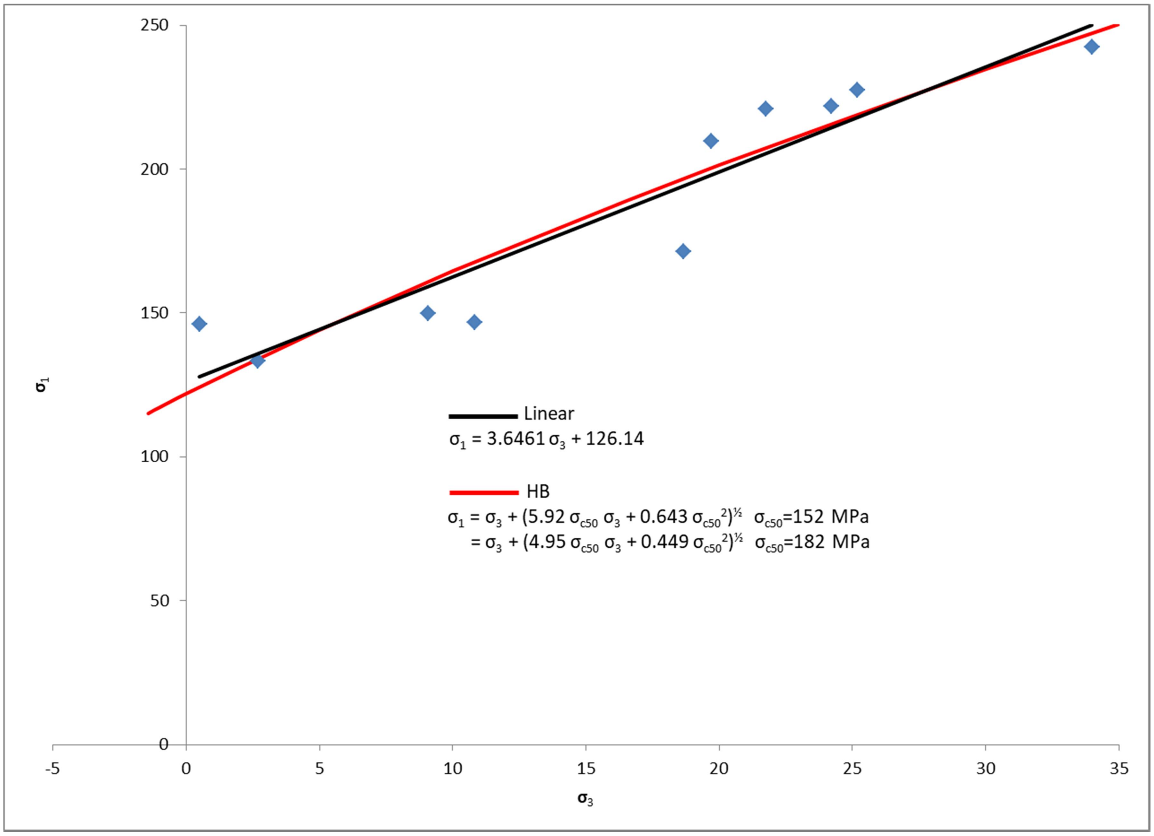




For completeness, let's now find the best fit Hoek-Brown line $\sigma_1 = \sigma_3 + (m \times \sigma_{c50} \times \sigma_3 + s \times \sigma_{c50}^2)^{½}$ through the sill pillar calibration points. To do this I rearrange the Hoek-Brown criterion into a linear form as $(\sigma_1 - \sigma_3)^2 = m \times \sigma_{c50} \times \sigma_3 + s \times \sigma_{c50}^2$. Using linear regression it can be found that the best fit values are m = 5.92 and s = 0.643. Setting σ_{c50} = 182 MPa, and using linear regression, it can be found that the best fit values are $m \times \sigma_{c50}$ = 901.2 and $s \times \sigma_{c50}^2$ = 14863.

Setting σ_{c50} = 152 MPa, it can be found that m = 5.92 and s = 0.643. Setting σ_{c50} = 182 MPa, it can be found that m = 4.95 and s = 0.449. These lines are identical.

The standard deviation for this line can be calculated as ±14.5 MPa, and the coefficient of variation is ±7.7%. A insignificantly better fit than the straight line. This line is superimposed on the straight line below. It would give the same result as shown in the figures above.



250
200
150
100
50
0
-5
0
5
10
15
20
25
30
35
σ1
σ3
Linear
σ1 = 3.6461 σ3 + 126.14
HB
σ1 = σ3 + (5.92 σc50 σ3 + 0.643 σc50²)½ σc50=152 MPa
= σ3 + (4.95 σc50 σ3 + 0.449 σc50²)½ σc50=182 MPa