Elastic softening to simulate plastic yielding

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Abstract

Elastic modelling is often sufficient to solve most geomechanics problems, however there are cases where non-linear/yielding rock response is necessary to model actual in situ behaviour. Although this can be due to stresses exceeding the strength, it could also be due to naturally occurring weak ground zones, or a result of ground conditioning due to destress blasting, hydraulic fracturing or concentrations of seismicity. In all of these cases it is desired to simulate ground yielding and stress redistribution beyond what is provided by linear elastic models.

Plasticity modelling is one method of simulating yielding rock mass response. While undoubtedly the best approach currently available, this method is known to be complex and expensive. There are many issues with plastic modelling that can invalidate predictions made with the method. Inaccurate specification of the many input parameters and inherent flaws in the theoretical basis can result in models that are no better than the original elastic model and in fact can be less accurate and hence misleading.

Keep in mind that the objective here is to simulate ground yielding and stress redistribution effects. While on one hand elastic modelling is too limiting, on the other hand plastic modelling appears to be an overly complex and uncertain method to achieve this goal. In this article, material softening is proposed as a simplified alternative approach that can be considered. While certainly not any sort of replacement for plastic modelling, material softening can be used simulate ground yielding, stress redistribution and hence the possible impact on surrounding areas. The advantage of this approach is that it allows the engineer to investigate non-linear behaviour quickly and easily.

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Introduction

In many cases, a well calibrated elastic model can adequately represent actual in situ behaviour sufficient for the purposes of engineering design. However, a shortcoming of elastic modelling is that the ground can become overstressed and hold too much load (refer to *Elastic Response* in Figure 6) when in reality it should yield and shed load to adjacent abutting areas, i.e. de-stress (refer to *Plastic Response* in Figure 6). Put into context, the first and foremost objective of any non-elastic modelling is simply the reduction of stress, in for example over-loaded pillars. Care must obviously be taken to de-stress by a realistic amount consistent with actual in situ behaviour. If this later requirement is not satisfied properly, then the results of any non-elastic modelling are not necessarily better than the original elastic model and in fact can be less accurate and hence misleading.

One method of “de-stressing” the rock is to use plastic modelling. With this method, the amount of destressing is controlled by ensuring that the final stress state in the affected pillar satisfies a well calibrated strength criterion. Although at first glance this sounds straight forward, it is not easy to arrive at an accurate estimate of strength on a rock mass scale. In addition, plasticity flow rules require specification of a large number of indeterminate input parameters such as peak, residual, softening and dilation effects. Accurate specification of these many parameters is required in order to drive the plasticity flow rule. In addition to this, there are also poorly understood flaws inherent in plasticity theory (see “Issues inherent to plasticity modelling theory” section below for details) due to shear-bands that form during strain softening. This behaviour can make reliable calibrations impossible and often invalidates predictions made with this method.

Although there is a justifiable desire to conduct a more realistic simulation than can be provided by elasticity, plasticity seems to be an overly complex, difficult and expensive approach to achieve this. Plastic analysis is well known to be a difficult exercise since is requires the use of advanced modelling techniques as well as extensive user knowledge and skills, excessive amount of time and expense. Uncertainty in both the input parameters and the underlying theoretical basis often provides predictions with little reliability. Plasticity appears to be an undesirably complex approach when contrasted with the simple desire for stress redistribution,

An alternative method for “de-stressing” the rock is to simply soften the ground in the overstressed zones. It is felt that softening would be useful for simulation of stress redistribution due to the presence of yielding zones. Although yielding could be a result of excess loads, it could also be due to naturally occurring weak ground zones, or a result of ground conditioning due to destress blasting, hydraulic fracturing or concentrations of seismicity. Softening offers a simplified and hence a desirable alternative to plasticity. This process is relatively simple, stable and no more computationally intensive than elasticity. Obviously it will be necessary to carefully control the amount of softening, and hence de-stressing, such that model behaviour is consistent with actual in situ behaviour.

Material softening offers a method to de-stress situations such as: pillars, hydraulically fractured zones or pre-conditioned ground. This would also be useful for simulations of weak contact zones, especially when they lie along exposed hanging walls. Done correctly, softening offers a more realistic simulation of rock mass behaviour than elastic modelling at little extra analysis cost. This also offers a simplified route to investigate the possible effectiveness of ground conditioning whether done with explosives or with hydraulic fracturing, and also damage due to accumulating seismicity. As with all numerical simulations, calibrations are required to determine the correct amount of softening required to best match historically observed behaviour.

Tunnel wall yielding example

To demonstrate the simplicity of the softening procedure, an example of an overstressed tunnel with side wall yielding is illustrated below (Figure 1). Note that the objective of this exercise is not to duplicate results from plastic modelling, nor should one expect to be able to do so. However, this will demonstrate side wall softening, de-stressing, stress transfer and yielding. The theory behind softening is explained in detail in subsequent sections below. Here it is desired to demonstrate that with softening:

- stresses can be dissipated to surrounding abutments,
- larger strains and displacements associated with plasticity can be generated.

Consider a 1 m radius circular tunnel with 30 MPa homogeneous pre-mining stresses. A Mohr-Coulomb failure criterion has been used throughout with a UCS of 11.9 MPa, friction angle of 30°, Young's modulus of 6780 MPa and Poisson's ratio of 0.21 This model has been analysed using Map3D Visco-Plastic. Elastic/perfectly plastic behaviour has been used with dilation rate set to zero. The major principal stress and displacement are shown below in Figure 1Figure 20.

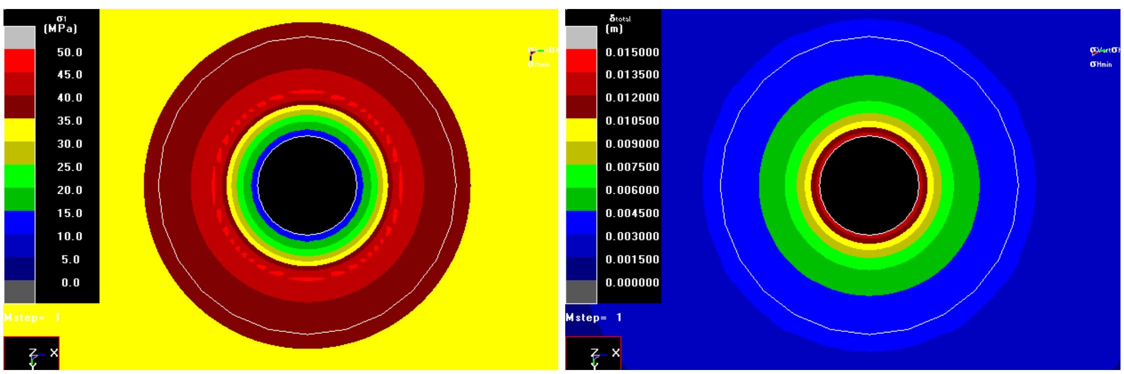

Figure 1: Major principal stress and displacement for plastic tunnel model

These model results closely match the closed form analytic solution for the plastic stresses and displacement as shown in Figure 2. For reference, the elastic analytic solution is also shown.

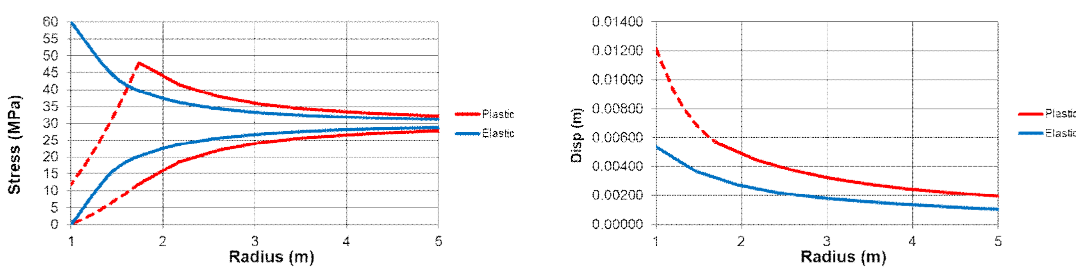

Figure 2: Principal stresses and displacement for plastic and elastic analytic solutions

In order to model the above with softening, a zone around the side walls of the tunnel has been softened as shown below (Figure 3) where the drive is shown in blue and softened zone is shown in purple. This softened zone has been chosen to be the same size as the plastic zone determined above (1.74 m radius).

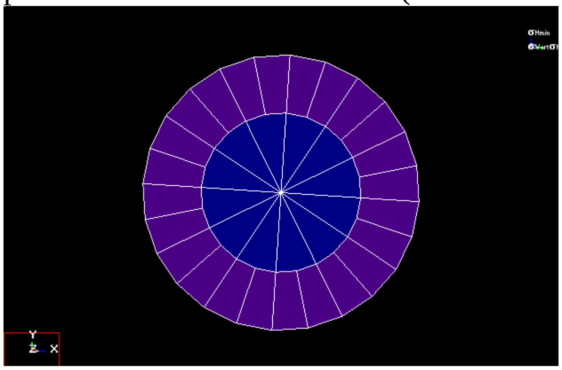

Figure 3: Model of tunnel with softened zone

In this case it is desired to demonstrate that plastic closure can be emulated with a simple softened model.

In this case, the elastic shear modulus was softened by a factor of 0.2, while the bulk modulus was held constant. The stresses and displacements for the softened model are shown in Figure 4 and labelled as "Softened" in Figure 5.

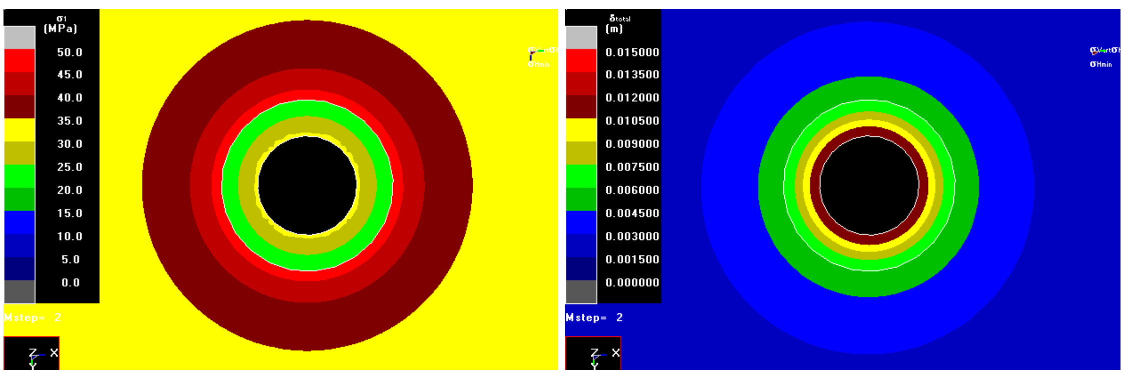

Figure 4: Major principal stress and displacement for softened model

The softened results shown in Figure 5 show an excellent match for both the plastic minor principal stress and displacement. Note that while the actual softened major principal stress values do not match the plastic results in the yield zone in detail, on average they are about the correct magnitude. Also the stress has been redistributed to the elastic "abutment" quite accurately showing nearly the exact amount of stress concentration expected.

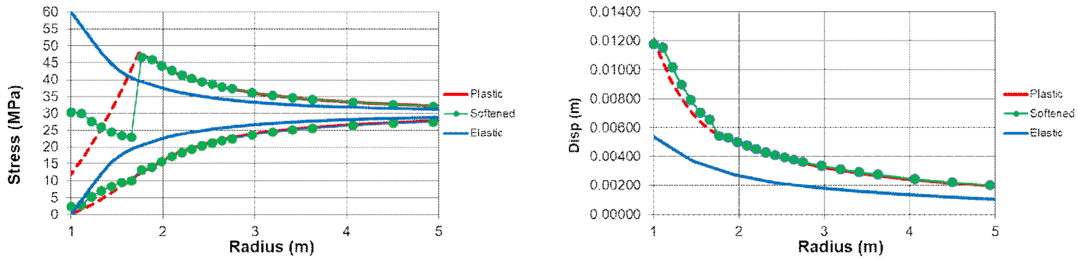

Figure 5: Principal stresses and displacement for softened model

While the results for softening do not match the plastic analysis results perfectly, they have obviously simulated yielding of the side wall material and driven the elastic results much closer to those from the plastic model. It would appear that material softening can be used to simulate ground yielding, stress redistribution and hence the possible impact on surrounding areas. The advantage of this approach is that it allows the engineer to investigate non-linear behaviour quickly and easily when compared to plastic modelling. Softened models require very little increased analysis time compared to simple elastic models.

Methodology for de-stressing in numerical modelling

In numerical modelling, de-stressing of an elastic rock mass is accomplished by allowing plastic strains, as shown in Figure 6. These strains act to dissipate the excess stress indicated by the elastic response.

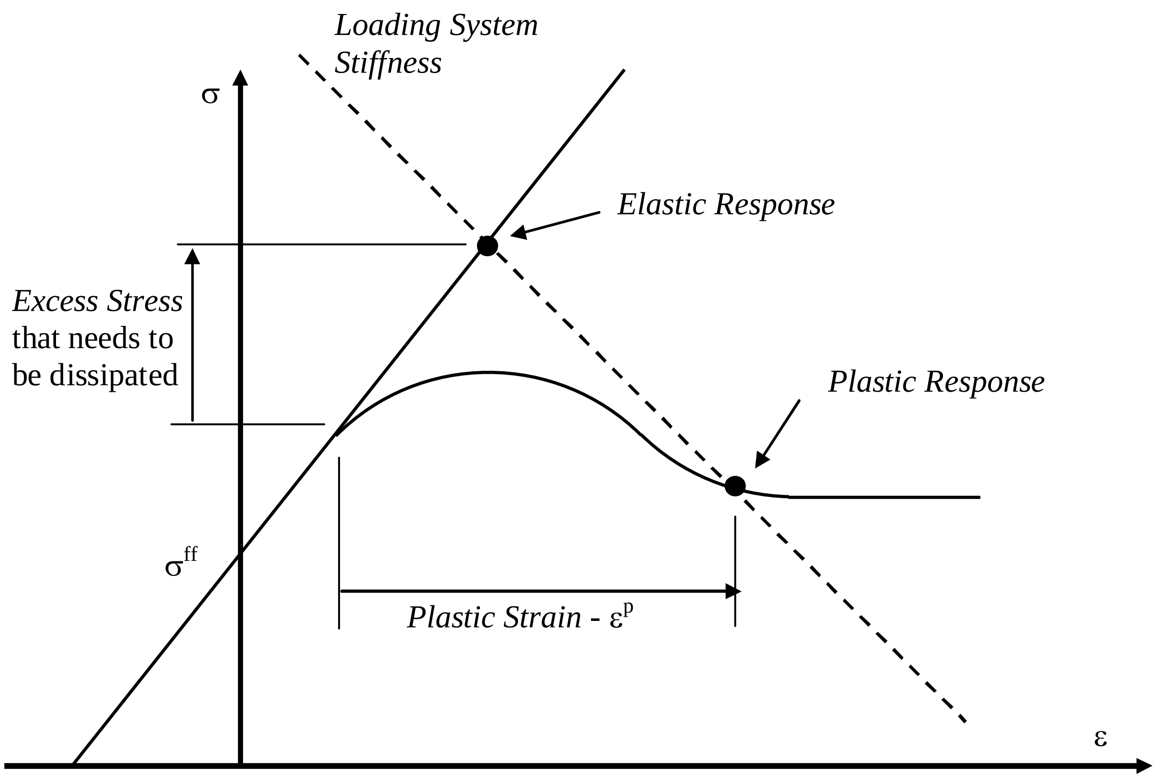

Figure 6: Elastic versus plastic response

Note that the *Excess Stress* resulting from elastic modelling can be related to the *Plastic Strain* determined from plastic modelling through *Loading System Stiffness*. Note that this figure is not meant to be technically accurate, but only to describe the elastic-plastic yielding process.

In current numerical modelling codes, de-stressing of an elastic rock mass can be accomplished in two different ways: incremental strain theory and total strain theory. Although both methods achieve the same end result, the former method has the distinct advantage of being easier to implement from a mathematical point of view. This readily allows for implementation of complex plasticity flow rules, and is hence the procedure of choice in most modern plasticity modelling programs. The later approach is conceptually easier to understand and is hence most attractive for hands on, user controlled situations such as softening.

Incremental strain plasticity theory

In plasticity modelling, shifting the elastic stress strain curve downwards is used to induce plastic strains. In Figure 7 shown below, increments ($\Delta\sigma^{\circ}$) of what are known as "initial stresses" are applied in the model. It can be observed that although technically this shifts the elastic stress-strain response downwards, this can be interpreted as being equivalent to increments of plastic strain (Δe^{p}) resulting in a reduction of the stress ($\sigma^{(1)}$,

$\sigma^{(2)}$, $\sigma^{(3)}$, $\sigma^{(4)}$…). Note that in this figure, the modulus has not changed and the same underlying elastic model still in use, but supplemented by initial stresses.

The objective of an automated plasticity model is to determine the amount of initial stress and hence plastic strain required to dissipate the elastic excess stresses such that the final stress state moves down to the plastic stress-strain response curve, and hence onto the strength envelope. In the example below it can be observed that as initial stress increments are applied, the stresses progressively reduce ($\sigma^{(1)}$, $\sigma^{(2)}$, $\sigma^{(3)}$, $\sigma^{(4)}$…). In the case depicted, approximately 8 increments of initial stress are required to achieve the value $\sigma^{(8)}$ which is on the stress strain curve.

Since both the excess stress and the loading system stiffness will vary from location to location, varying amounts initial stress must be applied at many different locations throughout the rock mass in order to satisfy the flow rule. This requires discretization of the solid rock mass into many smaller zones. Iterative solution procedures are used to find the required initial stress values in each zone and thus satisfy the failure criterion at all of these locations. It is the combined requirement of having to solve equations throughout the rock mass and the use an iterative process that makes plasticity analysis so computationally demanding when compared to elastic analysis.

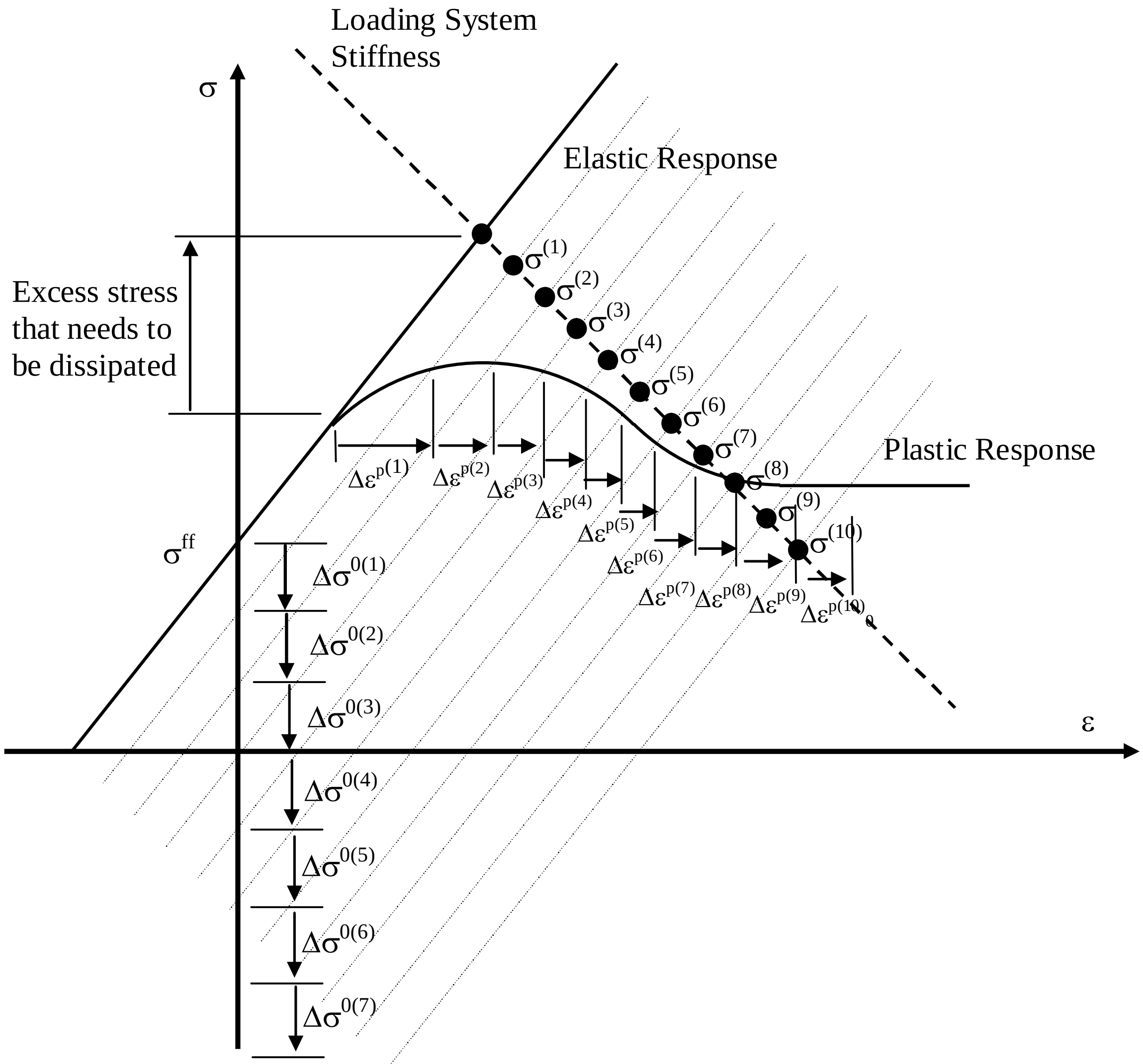

Figure 7: Incremental strain plasticity theory

Note the behaviour depicted in Figure 7 is not meant to be technically accurate, but only to describe the elastic-plastic yielding process. In reality the complete stress and strain tensor must be accounted for. In addition to satisfying the strength criterion, strain softening and volume changes (dilation or compaction) also need to be controlled. This will require the application of different amounts of initial stress at various orientations (a complete tensor of initial stress components). Complications arise when for example plasticity drives the magnitude of σ_1 below the magnitude of σ_2 and when dealing with tensile conditions. Well formulated flow rules can accommodate these conditions, but at the expense of being complicated and requiring many input parameters.

Total strain plasticity theory

An alternative method to induce plastic deformations is shown in Figure 8. Here, the stress-strain modulus is progressively reduced in magnitude. It can be observed that although technically this reduces the stiffness of the elastic stress-strain response, this can be interpreted as being equivalent to increments (Δe^p) of plastic strain.

As above, this method can be used with plastic modelling to dissipate the elastic excess stresses such that the final stress state is on the plastic stress-strain response curve, and hence on the strength envelope. In the example below it can be observed that as modulus is reduced, the stresses progressively reduce ($\sigma^{(1)}$, $\sigma^{(2)}$, $\sigma^{(3)}$, $\sigma^{(4)}$…). In this case approximately 6 increments of modulus reduction are required to achieve the value $\sigma^{(6)}$ which is on the stress strain curve, and hence on the failure envelope.

Since both the excess stress and the loading system stiffness will vary from location to location, varying amounts of softening will be required at different locations. As above, iterative solution procedures are used to find these values and thus satisfy the failure criterion throughout the rock mass. This is an alternate procedure for implementing plasticity that offers no particular advantage over the incremental strain plasticity theory. This procedure will result in exactly the same response obtained from incremental strain theory described in the previous section.

Note that this figure is not meant to be technically accurate, but only to describe the elastic-plastic yielding process. In reality the complete stress and strain tensors must be accounted for. Besides satisfying the strength criterion, strain softening and volume changes (dilation or compaction) also need to be controlled. Since this is considered to be somewhat more complex with this method when compared to the increment strain method, this is not the method of choice in most plasticity programs.

Note that with plasticity theory the softening process is automated whereby the flow rule and hence failure criterion are satisfied at all locations. Note that if desired this process can be controlled manually by the user. While it is not uncommon to think in terms of simply reducing Young’s modulus while holding Poisson’s ratio constant, this results in simultaneous reduction of both the shear and bulk moduli, This is because both the shear and bulk moduli are proportional to Young’s modulus. Although this has the desired effect of de-stressing, it is also accompanied by significant volume changes. This effect could possibly be desirable in simulation of for example backfill simulations, but it is not very representative for the case of rock mass yielding,

Rock failure is generally understood to be the result of shear failure and can be simulated with shear modulus reduction: whereas volume changes are associated with dilation or compaction depending on the stress conditions and nature of the rock itself. Shear modulus reduction can be used to reduce the stress differential (σ_1-σ_3), and bulk modulus reduction can be used to control the volumetric strain ($\varepsilon_{volume}=\varepsilon_1+\varepsilon_2+\varepsilon_3$). This allows independent control of the shear failure process and dilational response. Whether softening is automated or done manually, care must obviously be taken to de-stress by a realistic amount to ensure that the final stress state in the affected pillars satisfies a well calibrated strength criterion.

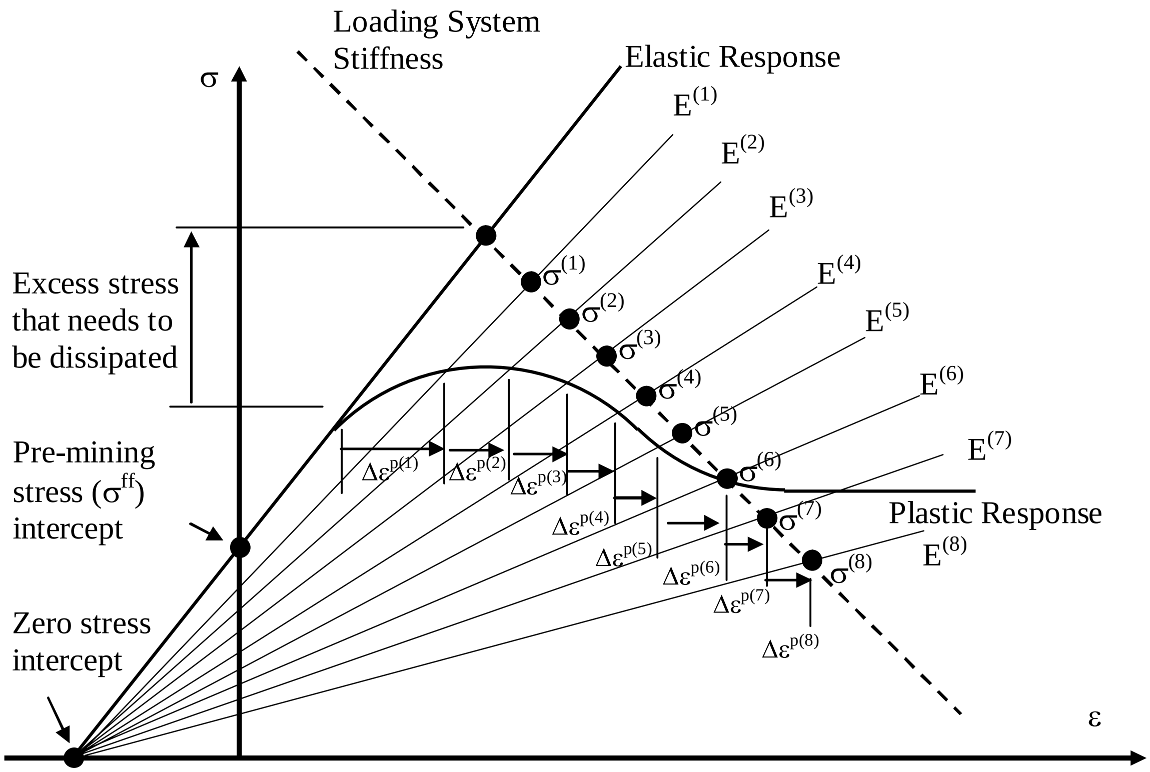

Figure 8: Total strain plasticity theory

In Figure 8, the correct way of softening is illustrated. Here the elastic stress-strain response with reduced modulus pivots around the zero stress intercept, not the pre-mining stress intercept. This is necessary since if you pivoted around the pre-mining stress intercept as shown in Figure 9, upon unloading you would attain a larger volume of material than you started with in clear violation of continuity requirements. In cases where the strength was below the pre-mining stress value, (e.g. $\sigma^{(8)}$) negative values of modulus would be required to sufficiently reduce the stress. The method illustrated in Figure 9 is clearly not acceptable.

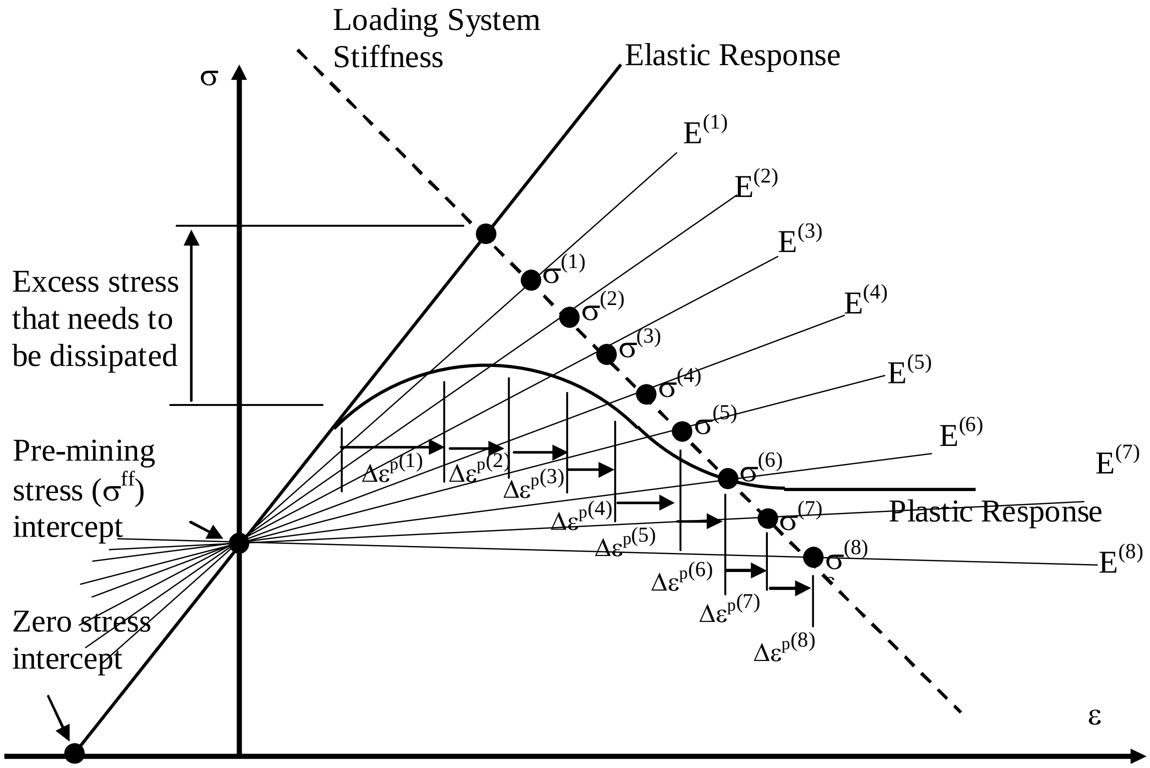

Figure 9: Total strain plasticity theory pivot

Simulation of de-stressed zones using the above methodology

The major advantage of incremental plasticity theory is that all aspects of the plastic strain tensor are easily controlled thus permitting incorporation of any desired degree of complexity in the flow rule (plastic stress-strain response curve). The resulting mathematical relations quickly become quite complex making this method unsuitable for manual control by the user. This method is however well suited to automated plasticity programs.

The major advantage of total strain plasticity theory is that when simplified to the concept of shearing and volume changes only, stress dissipation is easily controlled with only a very few parameters (namely reduction in both of the shear modulus and/or the bulk modulus). If the user's primary objective here is simply reduction of shear resistance (in for example highly loaded pillars or hydraulically fractures zones), the modulus reduction method offers an easy to understand procedure to achieve this. This method is therefore well suited to de-stressing manually controlled by the user. This can be readily accomplished from within elastic modelling programs. Keeping in mind that the primary objective of implementing plasticity is simply de-stressing, this provides a highly desirable capability since such programs are relatively simple, stable and are not any more computationally intensive then elastic modelling programs.

This method is not proposed as a replacement for plasticity modelling, but rather as a quick and easy method that can be used to investigate potential stress redistribution effects. Although one can argue that with this simplified procedure there is a potential loss in accuracy, one could also argue that this is also true of poorly calibrated plastic flow rules. In either case, the success of any implementation depends entirely on the care taken to de-stress by a realistic amount to ensure that the final result is representative of actual observed behaviour.

Plastic analysis has many shortcomings, but there is no doubt that it is a very capable technique. If it was calibrated properly then there is little doubt that it would give better predictions then you could ever hope to manage with softening. The point is that the proper calibration of a plasticity model is rarely if ever carried out. Although softening offers a less capable method, it is a method that is much easy to calibrate since it really has only one or two parameters to be concerned with. Engineers should consider whether they would rather have a well calibrated and thoroughly tested simple model or a poorly calibrated and untested complex model. With a well calibrated simple model you know how right or wrong you are likely to be, ie. known reliability. With a poorly calibrated or unverified complex model you don't even know if you are right or wrong or how wrong, ie. little or no reliability.

De-Stressing of a loaded pillar example

To demonstrate the simplicity of the softening procedure, an example of an overstressed pillar is illustrated below. Consider a 1x1 pillar between two drives shown in Figure 10. Here the pre-mining stresses have all been set to 30 MPa. The UCS of 25 MPa and friction angle of 30° has been used throughout. The plastic dilation rate is set to zero.

Note that the objective of this exercise is not to duplicate results from plastic modelling, but rather to demonstrate pillar softening, de-stressing and hence yielding. It is desired to demonstrate that with softening:

- pillar stresses can be dissipated to surrounding abutments,
- larger strains and displacements associated with plasticity can be generated.

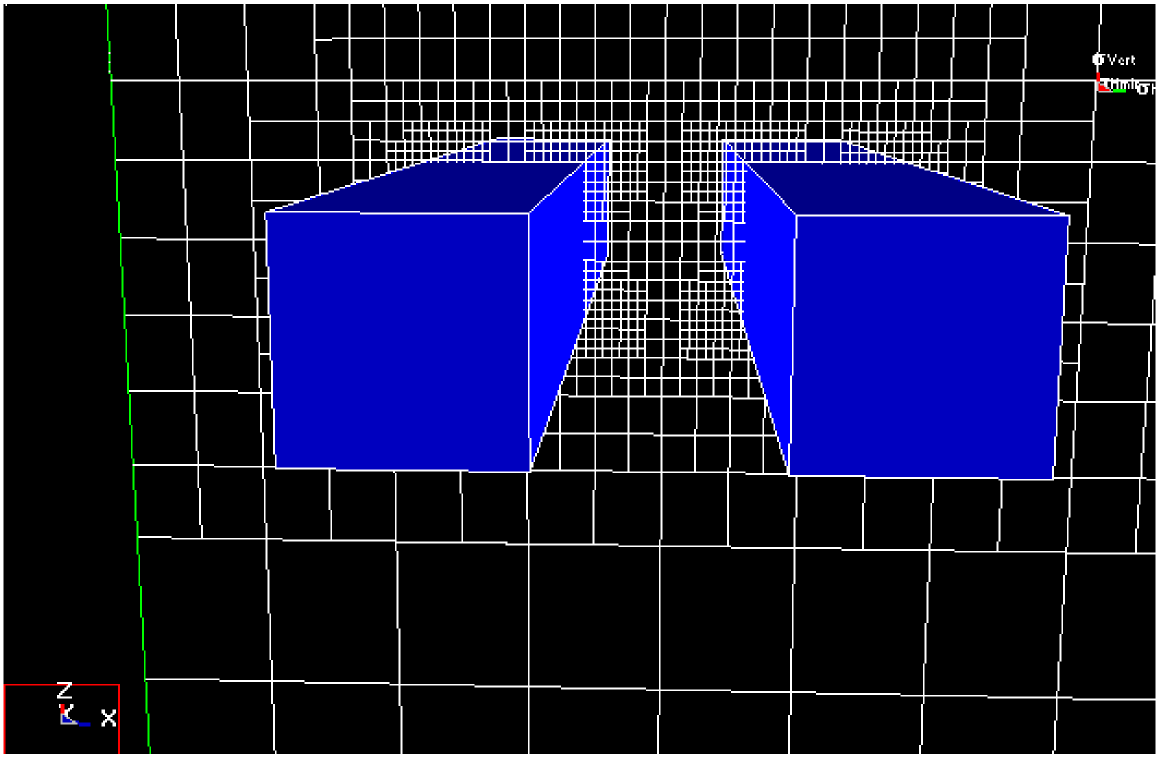

Figure 10: 1x1 Pillar Between 2 Drives

In Figure 11, the shear modulus *G* of the pillar material is progressively softened (using the total strain plasticity theory procedure outlined in Figure 8) by the factor *fg* such that the softened value equals $fg \times G$. Here *fg* varies progressively from 1 to 0. Note that with *fg* equal to 1, the results are exactly equivalent to an elastic model without any softening. With *fg* equal to 0 the results are for an elastic model without any shear resistance. In this later case no shear stresses can be sustained, hence $\sigma_1 = \sigma_3$.

If desired, the bulk modulus *B* of the pillar material could be softened as well by the factor *fb* such that the softened value equals $fb \times B$. In this case the bulk modulus is held constant throughout be specifying *fb* as 1.

For comparison, the results for a fully plastic model are shown and labelled *PM* in all figures. In this plastic model the dilation has been set to zero (the plastic volumetric strain component is zero).

If can be observed that as the pillar softens, σ_1 decreases and σ_3 increases. In Figure 11, σ_1 (at the centre of the pillar) falls below the strength envelope when *fg* reduces to a value of 0.55 The horizontal lines (labelled *s1 – PM* and *s3 - PM*) represent results from a fully plastic analysis for the same pillar model. Note that for this softening model, σ_1 never reaches the stress level from the plastic model.

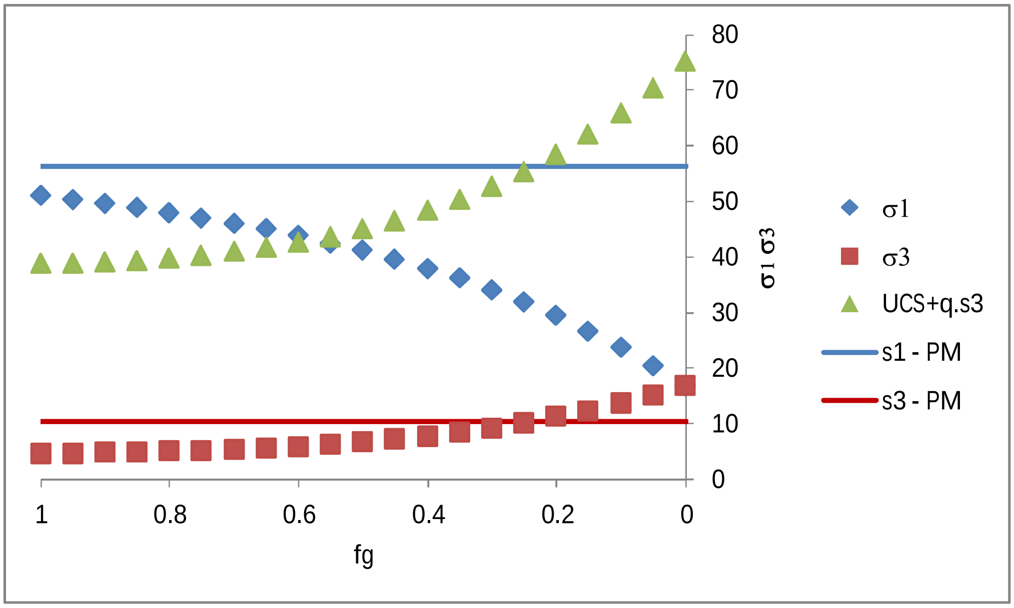

Figure 11: Stress Changes with Shear Modulus Softening – Centre Point of Pillar

Results for the same pillar are shown in Figure 12, but this time for a point at the side wall of the pillar. σ_1 falls below the strength envelope when *fg* reduces to a value of 0.45 Since this is the side wall, σ_3 is always equal to zero.

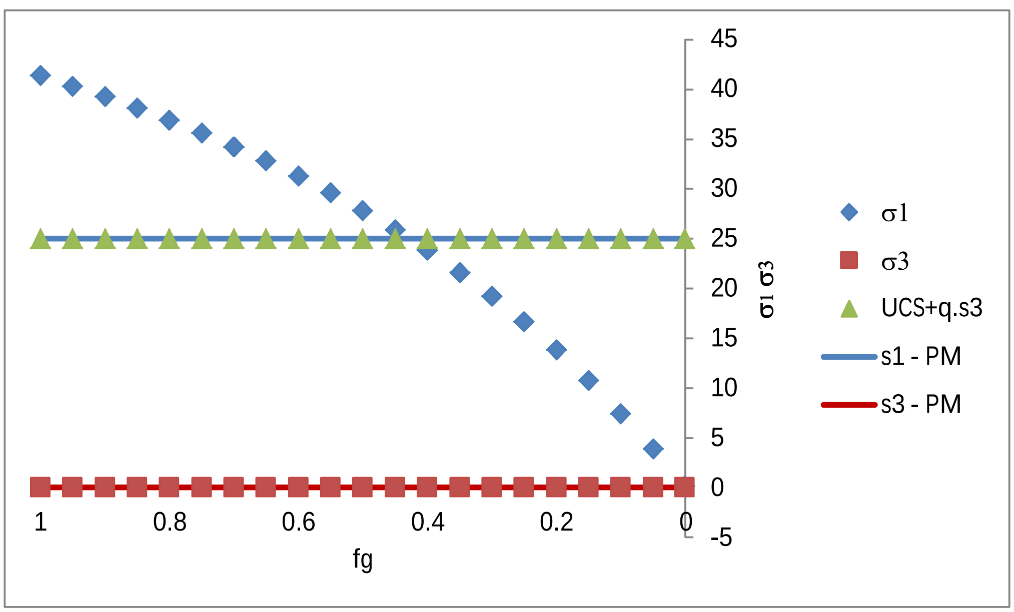

Figure 12: Stress Changes with Shear Modulus Softening – Side Wall

While the above stresses do not appear to be very accurate, keep in mind that this pillar was modelled using only one softened hex zone, with the same amount of softening across the pillar width. Whereas the plastic model used 600 hex zones. More accuracy could be obtained by using multiple softened zones each with different amounts of softening. Evidently, less softening is required near the centre than near the edges of the pillar. Recall that the objective of this exercise is not to duplicate the detailed results from plastic modelling, but rather to demonstrate the effectiveness of pillar softening to induce yielding of the pillar, cause increased strains and displacement, and hence redistribute stress to adjacent abutments.

In Figure 13, results for the strains at the centre point of the same pillar are shown. The maximum strain reached in the softened model is 1250, whereas from the plastic model the strain reaches 1800. The horizontal lines (labelled *e1 – PM*, *e3 - PM* and *evol - PM*) represent results from a fully plastic analysis for the same pillar model. The total strain (elastic plus plastic) is shown in all figures. As above, while this does not appear to be very accurate, keep in mind that this pillar was modelled using only one softened hex zone, whereas the plastic model used 600 hex zones. Some elastic volumetric dilation has occurred in spite of holding the bulk modulus at a contact value. This can attributed to the general reduction in the mean stress.

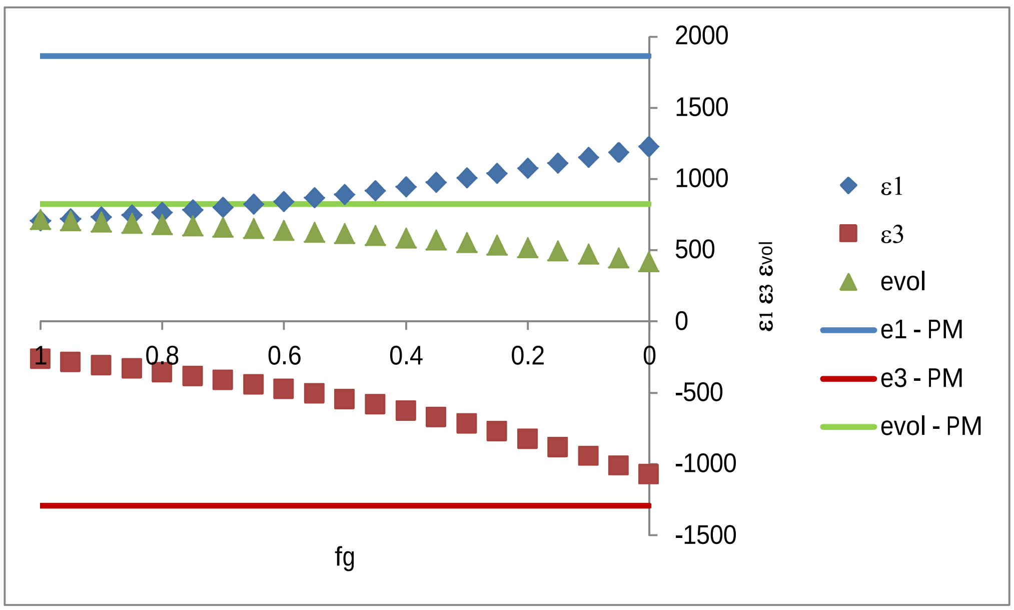

Figure 13: Strain Changes with Shear Modulus Softening – Centre Point

Results for the strains are shown in Figure 14, but this time for a point at the side wall of the pillar.

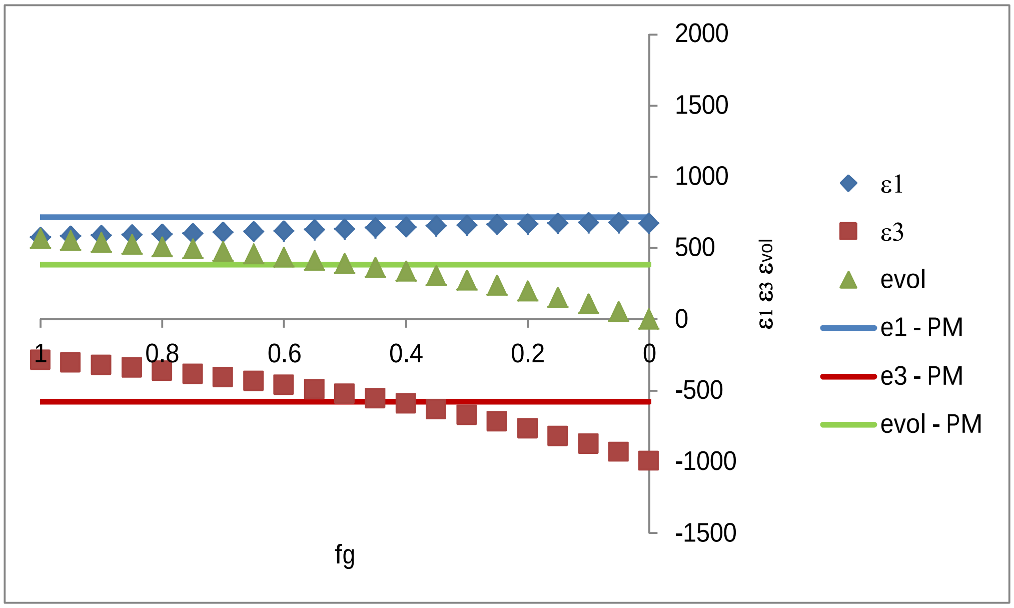

Figure 14: Strain Changes with Shear Modulus Softening – Side Wall

Results for the pillar closure (centreline displacement difference from top to bottom) and pillar dilation (centreline displacement difference from side to side) are shown in Figure 15. The horizontal lines (labelled *Closure – PM*, *Dilation - PM*) represent results from a fully plastic analysis for the same pillar model. Considering that this pillar was modelled using only one softened hex zone, whereas the plastic model used 600 hex zones, surprisingly the simplified softening model predicts the same amount of pillar closure and dilation as the much more complex plasticity model.

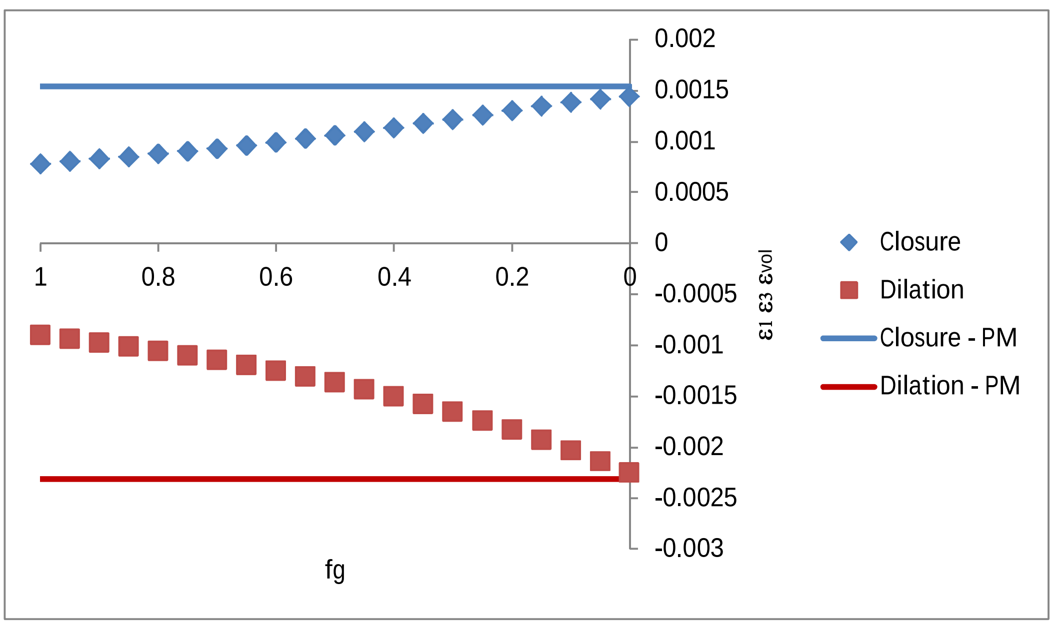

Figure 15: Pillar Closure and Dilation

Note that for the case of complete excavation of the pillar material, the maximum closure is 0.0051 m. In this case, softening has not drastically over or under estimated the response of the pillar, but has in fact provided quite a realistic response. Considering the simplicity of this procedure, this is a very positive outcome.

Contours of maximum principal stress (σ_1) in the softened elastic model versus the plastic model are shown in Figure 16. Although the abutment stress is very well represented, the stresses in the pillar do not compare very well. As discussed above, this is due to the use of a single value of softening for the entire pillar when in reality the centre area should soften less than the side wall area.

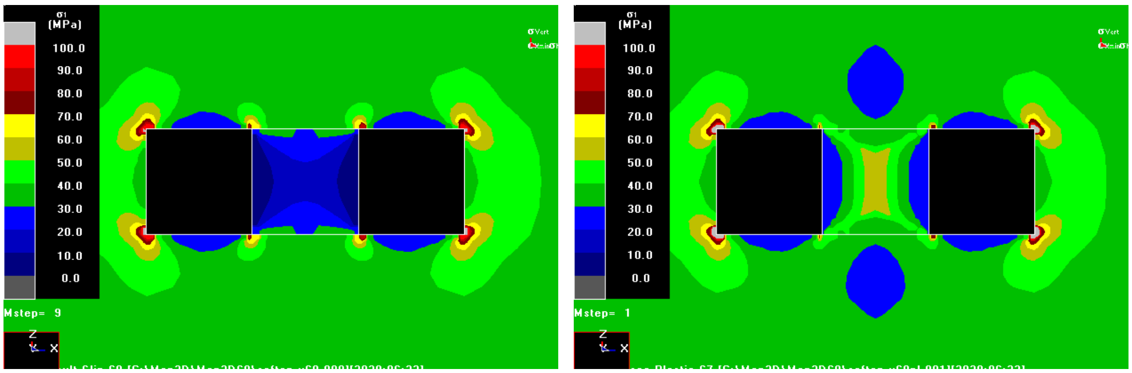

Figure 16: Maximum principal stress (σ_1) Contours (left - softened model, right– plastic model)

Contours of maximum principal strain (ε_1) in the softened elastic model versus the plastic model are shown in Figure 17. The softened elastic model shows surprisingly well defined shear bands.

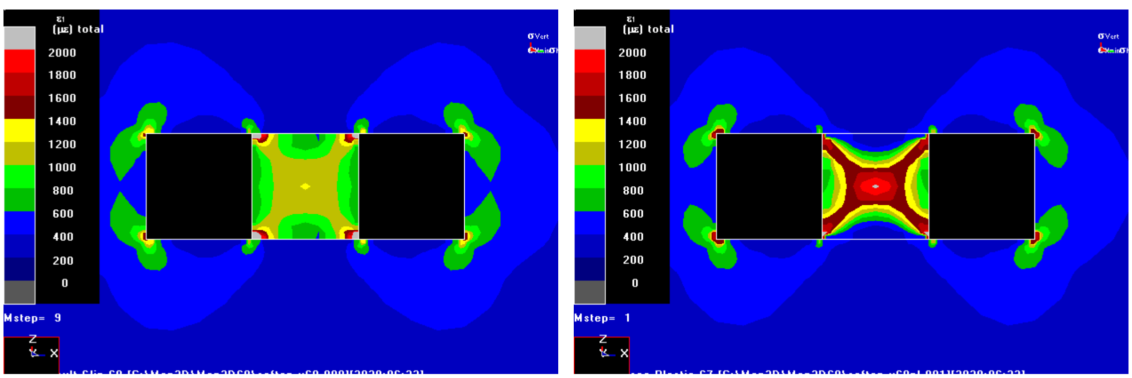

Figure 17: Maximum principal strain (ε_1) Contours (left - softened model, right– plastic model)

Contours of displacement in the softened elastic model versus the plastic model are shown in Figure 18. These results are surprisingly accurate considering that the softened pillar was modelled using only one softened hex zone, whereas the plastic model used 600 hex zones.

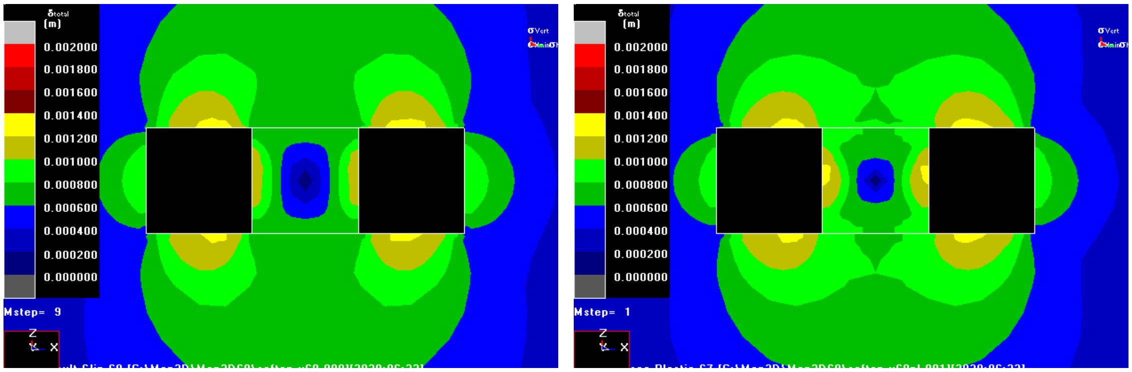

Figure 18: Displacement Contours (left - softened model, right– plastic model)

In all figures above, there was no volumetric softening (*fb* equals 1). Below in Figure 19, various amount of volumetric softening are shown for the case when *fg* equal to zero. Note that for volume softening with *fb* equal to 0.2 the results match the plastic model. For the sake of interest, results are also shown to demonstrate volumetric hardening (*fb*>1) as well.

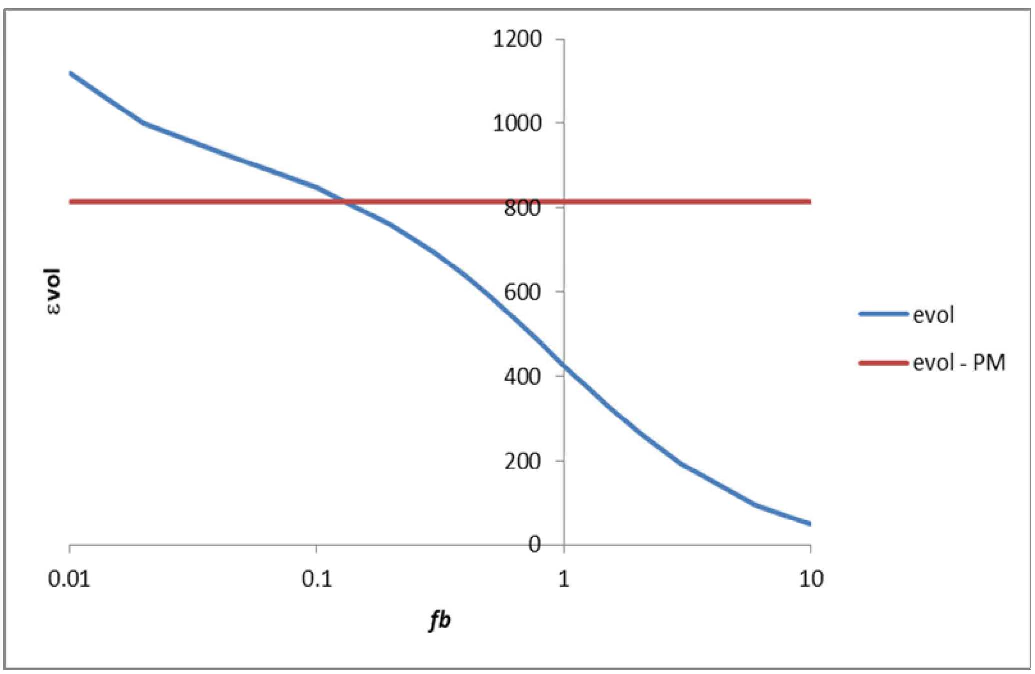

Figure 19: Volumetric strain ⅓(ε_1+ε_2+ε_3) for various values of volumetric softening – Centre Point

Drive wall yielding example

To demonstrate the simplicity of the softening procedure, an example of an overstressed drive with side wall yielding is illustrated below (Figure 20). Note that the objective of this exercise is not to duplicate results from plastic modelling, nor should one expect to be able to do so. However, this will demonstrate side wall softening, de-stressing, stress transfer and yielding. The theory behind softening is explained in detail in subsequent sections below. Here it is desired to demonstrate that with softening:

- stresses can be dissipated to surrounding abutments,
- larger strains and displacements associated with plasticity can be generated.

Consider a 1x1 drive with 30 MPa horizontal pre-mining stresses and 60 MPa vertical pre-mining stresses. A UCS of 50 MPa and friction angle of 30° has been used throughout. The strain for the plastic model is illustrated in the figure below. This model has been analysed using Map3D Visco-Plastic. Elastic/perfectly plastic behaviour has been used with dilation rate set to zero.

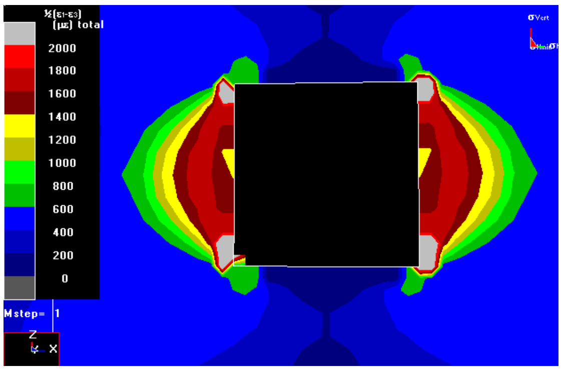

Figure 20: Maximum shear strain ε_{max}=½(ε_1- ε_3) for plastic model

In order to model the above plasticity with softening, two zones along the side walls of the drive have been softened as shown below in Figure 21. The drive is shown in blue and softened zones are shown in green.

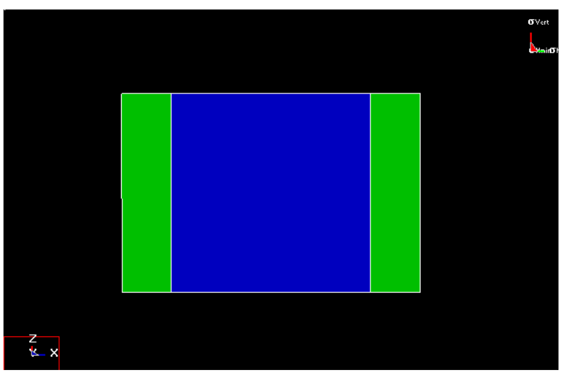

Figure 21: Model of drive with flanking softened zones

In this case it is desired to demonstrate that plastic drive closure can be emulated with a simple softened model.

The displacements for the plastic model are shown in Figure 22 and labelled as “Plastic” in Figure 23. For reference, the elastic analysis result is labelled as “Elastic” in Figure 23.

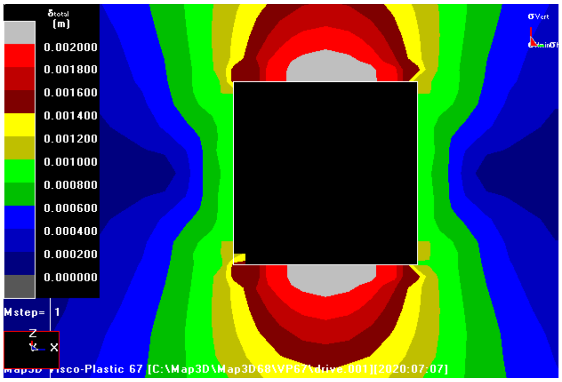

Figure 22: Displacement from plastic model

First consider the vertical closure in the drive as shown in the figure below. It would appear that softening with *fg* equal to 0.5 or less is required to get the softened model to deform the same as the plastic model. The exact amount is dependent on the volume softening. In this figure, 3 different values of volume softening are presented. *fb*=1, or no volume softening. *fb*= *fg*, or volume softening equal to shear softening. And finally (1-*fb*)=½(1-*fg*) which sets the volume softening (bulk modulus reduction) to half as much as the shear softening (shear modulus reduction).

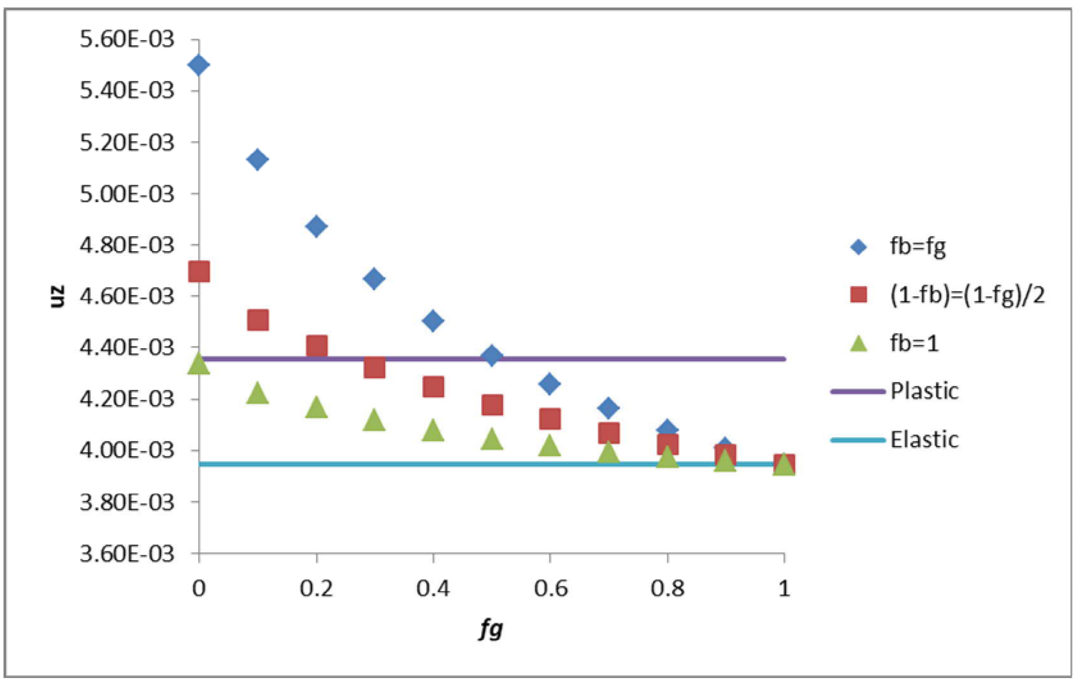

Figure 23: Vertical closure

Now consider the horizontal closure in the drive as shown in the figure below. It would appear that softening with *fg* equal to 0.3 or less is required to get the softened model to deform the same as the plastic model. The exact amount is dependent on the volume softening.

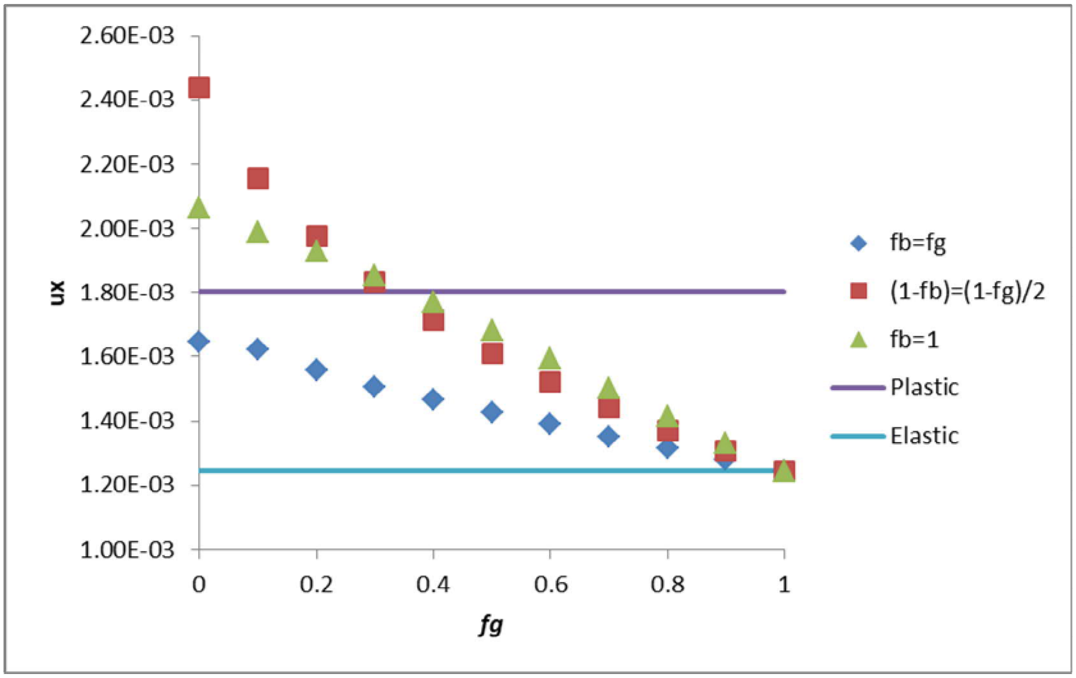

Figure 24: Horizontal closure

Taken together it seems that the best solution is obtained with *fg* equal to about 0.3 and *fb* set such that (1-*fb*)=½(1-*fg*) or *fb*=0.65 Set in this way, volume softening (bulk modulus reduction) is half as much as the shear softening (shear modulus reduction). This combination gives a similar amount both vertical and horizontal closure as the plastic model.

Results below are presented in sets of 3, with elastic results on the left, softened results in the centre and plastic modelling results on the right. The softened results illustrated are for the case with *fg*=0.3 and *fb*=0.65 Figure 25, Figure 26 and Figure 27 illustrate respectively major principal stress σ_1, maximum shear strain $\varepsilon_{max}=½(\varepsilon_1- \varepsilon_3)$ and volumetric strain $\varepsilon_{vol}=⅓(\varepsilon_1+\varepsilon_2+\varepsilon_3)$.

Figure 25 illustrates that the softened zones transfer the elastic stress out and away from the side walls in a manner similar to the plastic analysis.

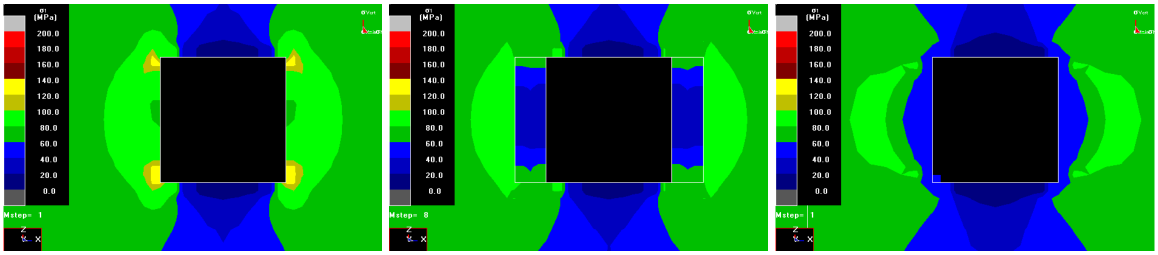

Figure 25: Major principal stress σ_1 - (left to right: elastic-softened-plastic)

Figure 26 illustrates that the softened zones induce large sidwall strains similar in magnitude to those determined with the plastic analysis.

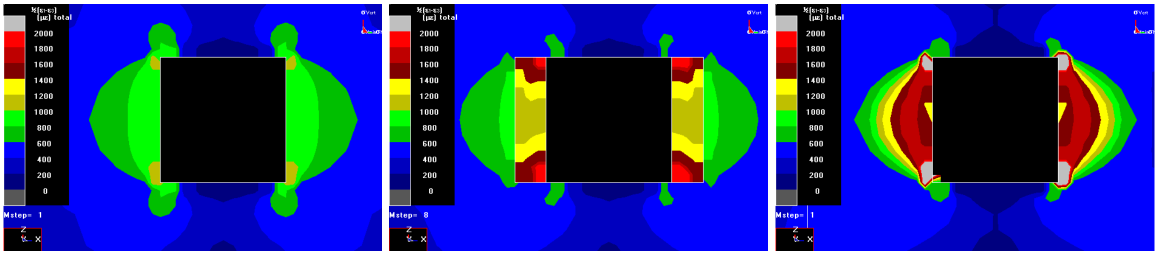

Figure 26: Maximum shear strain $\varepsilon_{max}=½(\varepsilon_1 - \varepsilon_3)$ - (left to right: elastic-softened-plastic)

Figure 27 illustrates that the softened zones induce volumetric strain in the side walls similar in magnitude to that determined with the plastic analysis.

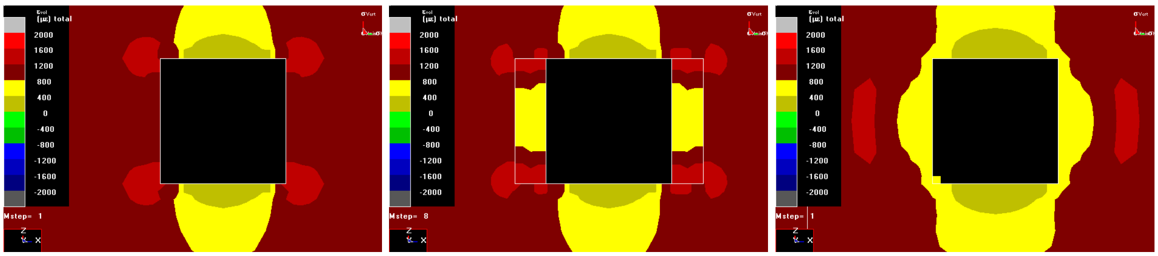

Figure 27: Volumetric strain $\varepsilon_{vol}=⅓(\varepsilon_1+\varepsilon_2+\varepsilon_3)$ - (left to right: elastic-softened-plastic)

Figure 28 illustrates that the softened zones induce horiztonal closure similar in magnitude to that determined with the plastic analysis.

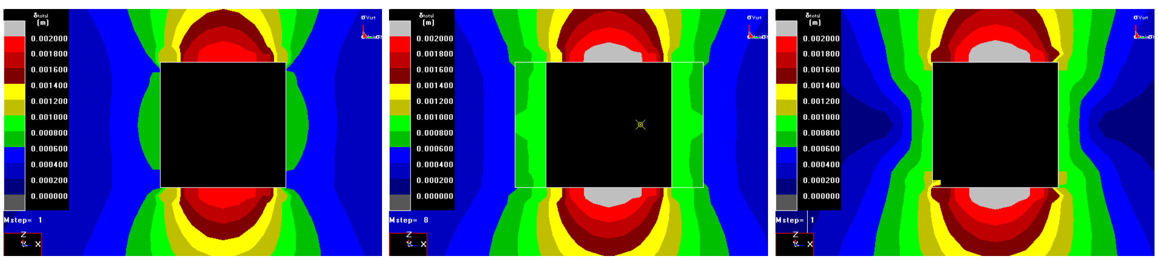

Figure 28: Displacement - (left to right: elastic-softened-plastic)

While the results for softening do not match the plastic analysis results perfectly, they have obviously simulated yielding of the side wall material and driven the elastic results much closer to those from the plastic model. It would appear that material softening can be used simulate ground yielding, stress redistribution and hence the possible impact on surrounding areas. The advantage of this approach is that it allows the engineer to investigate non-linear behaviour quickly and easily.

How to set up a softening model in Map3D

The pillar model described above is easily set up in Map3D by assigning the central pillar as an alternate material zone shown in green below (Figure 29).

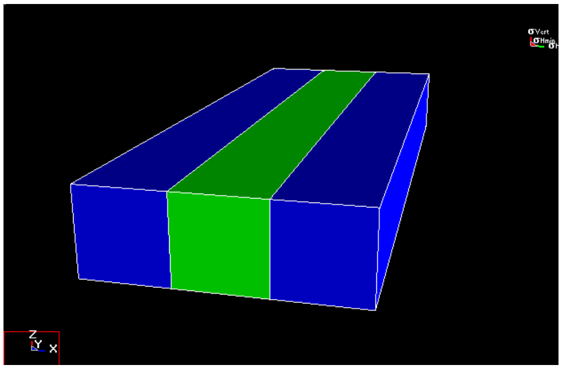
Figure 29: Softened pillar model

The shear modulus *G* of the pillar material is progressively softened from the host value to zero in a series of 11 steps. In order to accomplish this, the softening factor *fg* is progressively reduced from a value of one to zero such that the softened shear modulus value equals $fg \times G$. Here *fg* varies progressively from 1 to 0. Note that with *fg* equal to 1, the results are exactly equivalent to an elastic model without any softening. With *fg* equal to 0 the results are for an elastic model with no shear resistance. In this later case no shear stresses can be sustained, hence $\sigma_1 = \sigma_3$.

In this example, this is accomplished by setting up materials #10 through #20 each with different amounts of softening. These are assigned to the pillar block as follows (Figure 30)

Sort Steps	Sort Codes
Mining Step 1	20
Mining Step 2	19
Mining Step 3	18
Mining Step 4	17
Mining Step 5	16
Mining Step 6	15
Mining Step 7	14
Mining Step 8	13
Mining Step 9	12
Mining Step 10	11
Mining Step 11	10
Mining Step 12	
Mining Step 13	
Mining Step 14	
Close	Cancel

Figure 30: Pillar material assignments in Map3D (CAD > Entity Properties)

Note that the material numbers are set up in a descending sequence from 20 to 10. This is done intentionally since a descending sequence signals to Map3D that the same material persists from step to step, and only the material properties are to be updated. This can be compared to an ascending sequence of material numbers which would signal that a new material placement (eg. backfilling) is to take place: not desired here.

All material properties are initially set identical to the host material including the stress state. However each material is assigned softening values as follows (Table 1):

Material#	Softening factor *fg*
20	1
19	0.9
18	0.8
17	0.7
16	0.6
15	0.5
14	0.4
13	0.3
12	0.2
11	0.1
10	0.0

Table 1: Softening factor assignments

For example, material #15 appears as follows (Figure 31):

Figure 31: Material #15 showing 50% softening in Map3D (CAD > Properties > Material Properties)

This completes the set up required for this model.

Implementation notes and suggestions for other applications

There are many situations where elastic modelling is not considered adequate to provide accurate predictions. All of these situations are associated with non-elastic behaviour of the ground. For this reason, it is desired to incorporate non-elastic response into models.

Although plastic modelling offers a method of doing this, it seems to be an over complex procedure to achieve such a simple objective. As discussed above, there are many issues with plastic modelling that can invalidate predictions made with the method. Because of the complexity of plastic models, comprehensive calibrations against observed in situ behaviour is rarely if ever carried out. Inaccurate specification of the many input parameters and inherent flaws in the theoretical basis (see section below for details) can result in models that are no better than the original elastic model and in fact can be less accurate and hence misleading.

Since stress redistribution can also be achieved by material softening, this offers an alternative method to redistribute loads to surrounding abutments and requires only a few simple assumptions. This method is not proposed as a replacement for plasticity modelling, but rather as a quick and easy method that can be used to investigate potential stress redistribution effects. Obviously calibrations are required to determine appropriate amounts of softening, however the same can be said for plastic modelling. The difference is that in plastic modelling, there are many interacting complex parameters to consider. Whereas with softening, there are only a few input parameters (maybe just one, ie. shear softening, *fg*) to calibrate and one could expect back-analysis calibrations to be a much simpler and straight forward exercise. This makes comprehensive calibrations against observed in situ behaviour a much more straight forward exercise.

Plasticity analysis requires fine discretization of the rock mass if the plastic flow rule is to be satisfied throughout the failure zone. The modelling program iteratively adjusts the amount of plasticity so that the specified flow rule (strength equation) is satisfied at all locations.

The softening procedure proposed here does not require this: any enclosed shape will do fine, the limitation being that each zone is permitted to have one value of softening. The user is required to manually adjust the amount of softening. This requires some trial and error or other form of logic such as correlating the amount of softening to the density of hydraulic fracturing, intensity of de-stress/ground conditioning blasting, or accumulated seismicity.

Above, an example of softening of an overloaded pillar is represented as a demonstration of this methodology. As this is a novel concept, little or no experience is available on reliability or applications. However it is proposed that this method might be useful in many scenarios. This approach can be taken to set up softened zones for situations such as:

- seismically active faults,
- weak contact zones,
- soft or weak inclusions,
- hydraulic fractured zones,
- blasted destress/ground conditioned zones,
- seismically active zones,
- block cave zones,
- yielded remnant pillars.

In each case some trial and error or other form of logic would be required to determine representative amounts of softening. Although this has not been investigated at this time, it is likely that correlations can be established between the amount of softening and measurable parameters such as for example:

- density of hydraulic fracturing (more softening is expected with closer fracture spacing),
- intensity of de-stress/ground conditioning blasting (more softening is expected with increased charge density),

- a measure of accumulated seismic damage (more softening is expected with as magnitude and density of seismicity accumulates),
- volume drawn from cave draw point.

Hydraulic fracturing is often used as a method to condition high stress ground in an attempt to reduce the prevalence of seismicity or reduce the risk of rock bursts adjacent to mine workings. It is thought that by modelling these conditioned zones with softening (illustrated in Figure 32) could lead to a better understanding of how stresses are redistributed as a result of this process. It would interesting to evaluate how the potential for rock bursting changes both in the conditioned zone, but also in the areas adjacent to this. This could be evaluated both by examining the stress changes on nearby faults, and also by using energy release rate calculations.

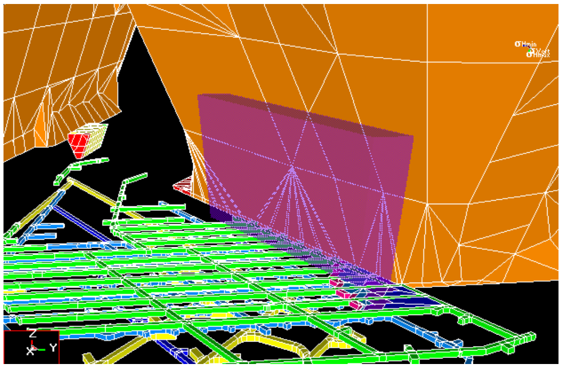

Figure 32: Shaded block shows softened hydraulically fracture zone

Ground conditioning by blasting ahead of drift rounds is often used as a measure to condition high stress ground in an attempt to reduce the prevalence of seismicity or reduce the risk of rock bursts in adjacent pillars and exposed faces. It is thought that by modelling these conditioned zones with softening (illustrated in Figure 33) could lead to a better understanding of how stresses are redistributed as a result of this process. Changes in the potential for rock bursting could be evaluated using energy release rate calculations.

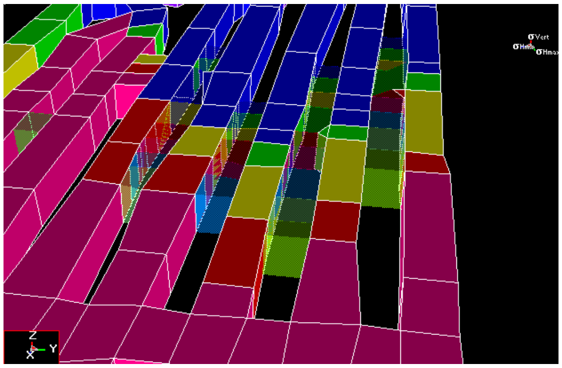

Figure 33: Shaded blocks show softened pre-conditioned pillars

Highly seismically active areas are obviously damaging the ground and shifting stresses to abuting areas. Softening (illustrated in Figure 34) could provide a method of simulting the redistribution of stress associated with this phenomenon.

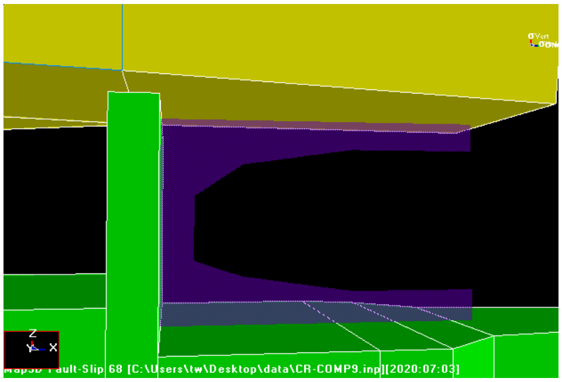
Figure 34: Shaded block shows softened seismically active zone

Weak hanging wall contacts are a common issue in mining. Plastic modelling of such zones is a complex exercise. Softening (illustrated in Figure 35) could offer a simple method of estimating the magnitude of displacements to be expected. This would be beneficial for evaluation of ground support design requirements.

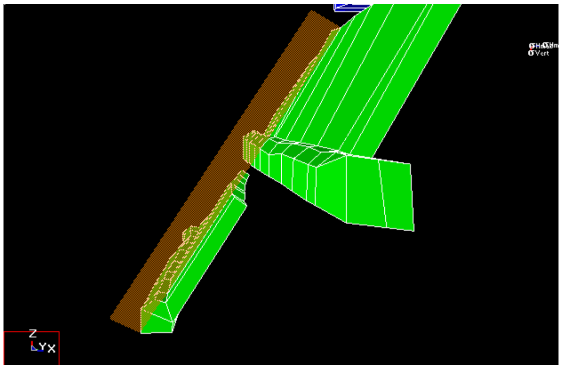
Figure 35: Shaded block shows softened weak hanging wall contact zone

Assigning different amounts of softening (illustrated in Figure 36) to zones above cave draw points could offer a simple method of estimating increased loading around draw points that have delayed extraction.

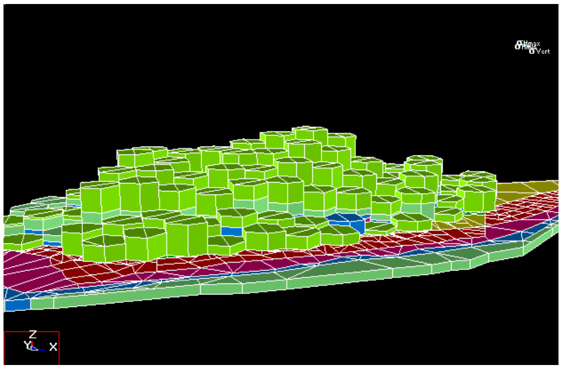

Figure 36: A model of cave zones above individual draw points

Older heavily mined zones are often included in models since it is anticipated that these zones are deformable and contribute significantly to stress redistribution. The simplest method for modelling these zones include simply assigning a fixed stress which is thought to be representative of combined effect of remnant pillars and filled or open excavations. A more accurate approach for modelling these zones is to simulate the yielded pillars, fill and excavation using plastic modelling. This later approach turns out to be such a large exercise that it is rarely adopted. Here it is suggested that a compromise approach would be to simply soften the zone (illustrated in Figure 37). Although the amount of softening would need to calibrated to obtain representative behaviour, both other approaches would require such calibrations as well. Softening offers a simplified but realistic approach in this case.

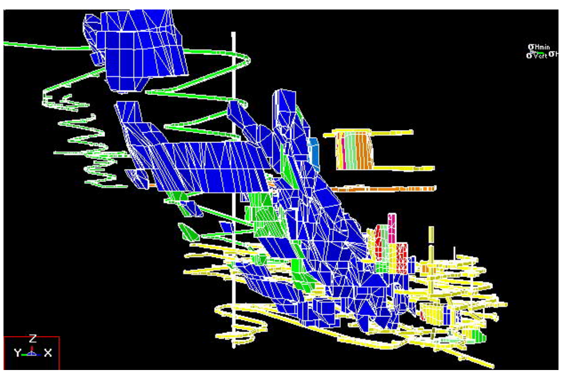

Figure 37: Older heavily mined zones (blue) could be simulated with softening

None of the above examples have been investigate in detail by the author at this time. These applications are proposed since it is thought that they would significantly enhance understanding of rock mass behaviour and provide alternative approaches to interpretation of observed behaviour.

Annex: Issues inherent to plasticity modelling theory

Plasticity is the method of choice to simulate the yielding and associated stress redistribution of rock. Although this is a popular method utilized in many state of the art modelling programs, there are many often insurmountable problems inherent with this method.

Plasticity theory is based on the concept of a flow rule that governs the inelastic behaviour. Inherent in this flow rule is the strength of the rock and a detailed mathematical description of how that changes with increasing strains and hence damage. The simplest possible flow rule is known as elastic-perfectly plastic (see Figure 38). This of course requires the specification of strength, including details about how to respond when under tensile stress conditions. Simultaneously, the dilation response must also be specified.

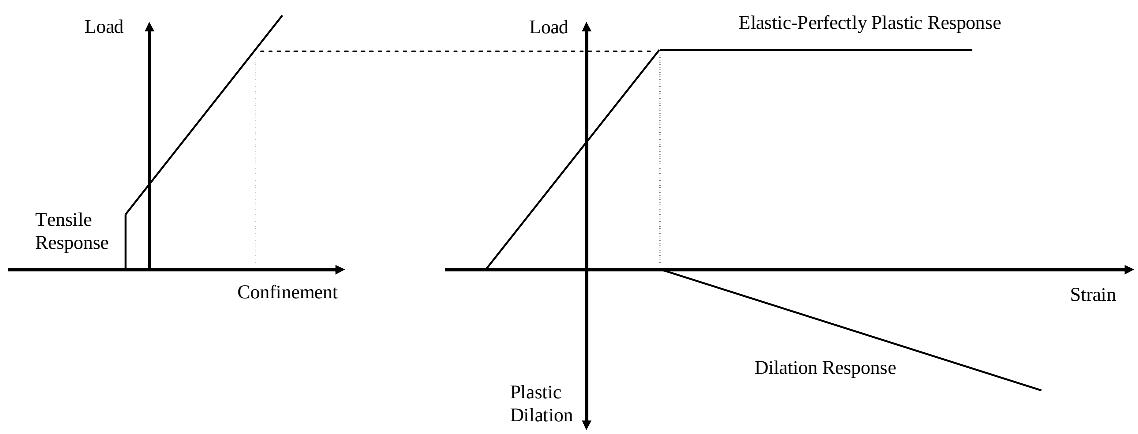

Figure 38: Simplified elastic-perfectly plastic response

Plasticity modelling theory issue #1– a large number of uncertain input parameters

Even this simplest of flow rules requires difficult behaviour assumptions and the specification of many input parameters which must be calibrated so that the model behaves in a manner representative of the actual rock mass. A minimum of 3 parameters are required to describe the strength envelope (eg. *UCS*, φ and tension cut-off). Assumptions must be made regarding whether or not there is any compressive strength when the tensile strength is exceeded. An additional 2 parameters are required to describe the dilation (a total of 5 parameters). It is common to just set the dilations to zero and ignore this part of the response completely. This does not make the issue go away.

In the simple model above, the strength does not change with advancing strains and hence damage, Most users of plasticity program are not satisfied with the simple behaviour depicted above and want to include some sort of strength degradation with damage in the form of strain softening. Hence it is more common to utilize a flow rule similar to that shown in Figure 39.

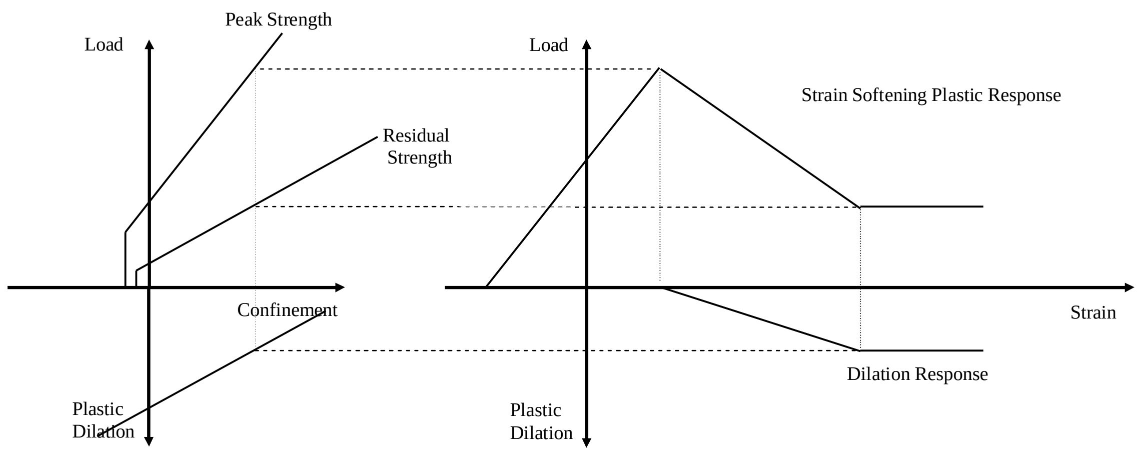

Figure 39: Strain softening plastic response

This specifies behaviour more similar to what is thought to be actual rock mass response. Here the strength reduces as the plastic strains accumulate. In addition to parameters identified above, additional parameters are now required to describe the residual strength (a minimum of 3 more parameters) and softening rate (1 more parameter). Note that the residual dilation is expected to be a strong function of confinement (a minimum of 2 more parameters).

Clearly the number of parameters (11 parameters at this point) is spiraling out of control. How is one ever going to be able to find sufficient back analysis examples to calibrate reliable values for so many parameters? Who has the motivation, time and resources to do a thorough calibration? In spite of these issues, it is not uncommon for plastic modellers to adopt even more complex flow rules requiring even more parameters in an attempt to make the flow rule appear more like what is thought to be actual rock mass response (smoother shaped response curves).

Clearly results from such models are being generated without being properly verified against field observations. Parameters are specified either by guesswork or based on laboratory tests. Although there are techniques to adjust laboratory test results to field scale, it has been the author’s experience that the variability of this degradation process is so large (on the order of ±50%) that the resulting parameters values have little or no certainty. Such values are used ignoring that fact that they have little or no demonstrable reliability, or at the very least unknown reliability.

Instead of opting for a highly complex flow rule with a large number of indeterminate parameters, one might be better off with a simpler flow rule that has less input parameters. In the later case it is more likely that thorough calibrations can be conducted and hence better model reliability can be establish.

Plasticity modelling theory issue #2 – non-unique input parameters

There is an inherent flaw in strain softening plasticity theory. To illustrate this, consider for example the simple crown pillar failure model illustrated in Figure 40.

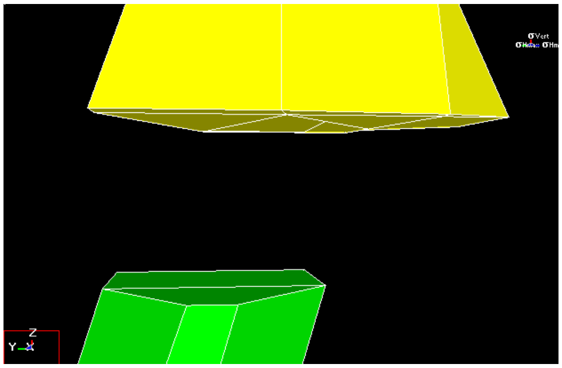

Figure 40: A simple crown pillar failure model

Plasticity models indicate that failure takes place by diagonal shearing through the pillar as shown in Figure 41 (these models have been analysed with the Map3D Visco-Plastic analysis program).

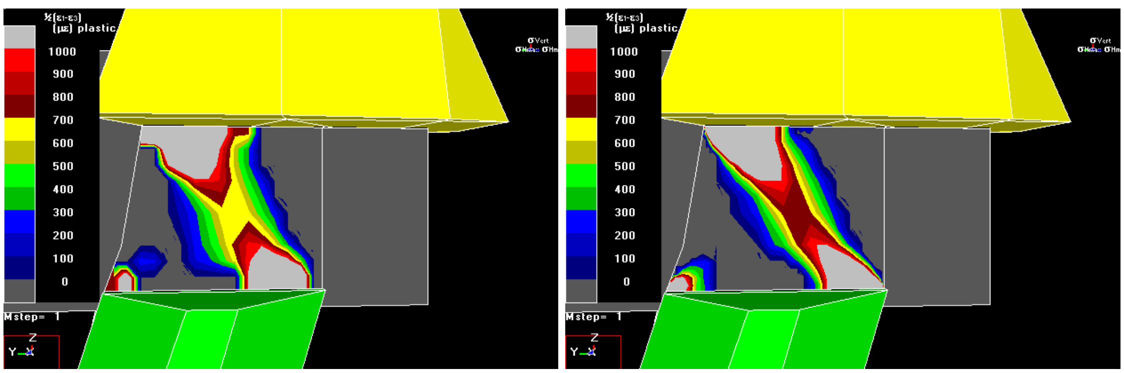

Figure 41: Diagonal shear band failure (left – course discretization, right – fine discretization)

Plasticity appears to make perfect sense until strain softening is introduced. The problem that arises is that the shear band width is a function of the discretization used. For course discretizations (left side of Figure 41) the shear band will be quite wide, whereas for very fine discretizations (right side of Figure 41) the shear band will be narrower. The shear band will always seek out a path that is a few discretization widths wide.

Note that in order to generate the same amount of displacement, a narrower shear band will require larger shear strains than a wider shear band. The problem is that with more shear strain, a finer discretization and hence narrower shear band will come to an equilibrated solution at a lower stress level than a courser discretization (see Figure 42). Hence upon failure, progressively finer discretizations will always predict lower pillar stresses, more stress redistribution to the abutments, and hence larger pillar closure (compare Figure 43 and Figure 44).

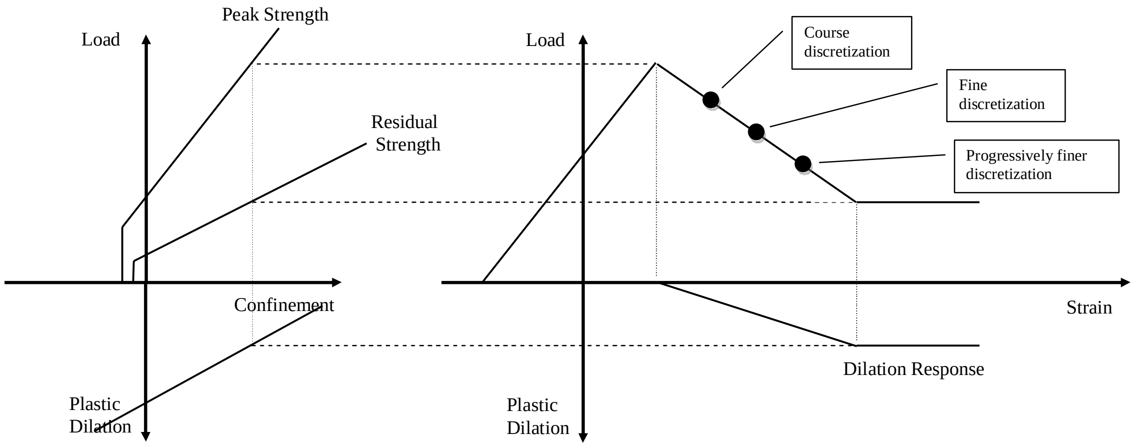

Figure 42: Strain softening plastic response

There is no point at which progressively finer discretiations will come to any sort of minimum shear band width since this is entirely dependent on the fineness of the disretization. Progressively finer discretizations will always seek out narrower shear bands and hence a minimum possible residual stress in the pillar.

In Figure 43 and Figure 44, stress and displacement changes induced by pillar yielding are illustrated. These are presented for the hangingwall/footwall oriented components of stress and displacement. These are calculated by subtracting the elastic modelling results (non-yielding pillar) from the plastic modelling results (yielded pillar).

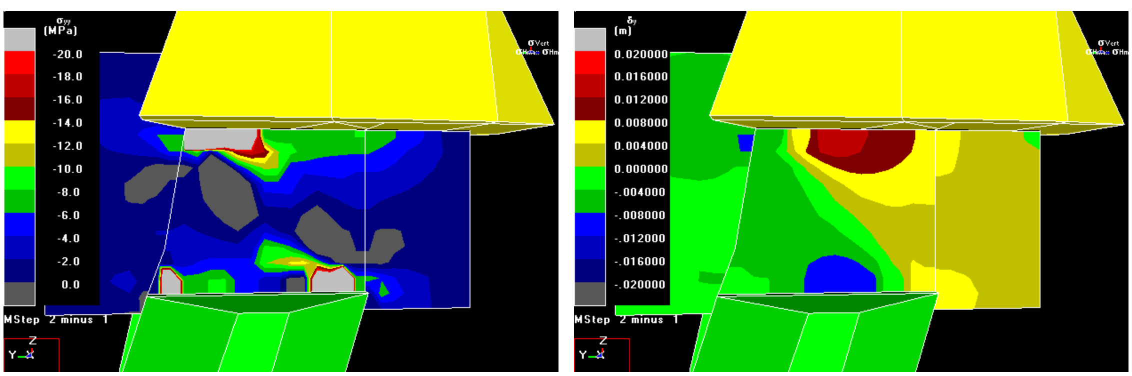

Figure 43: Stress (left) and displacement (right) change due to pillar yielding – course discretization

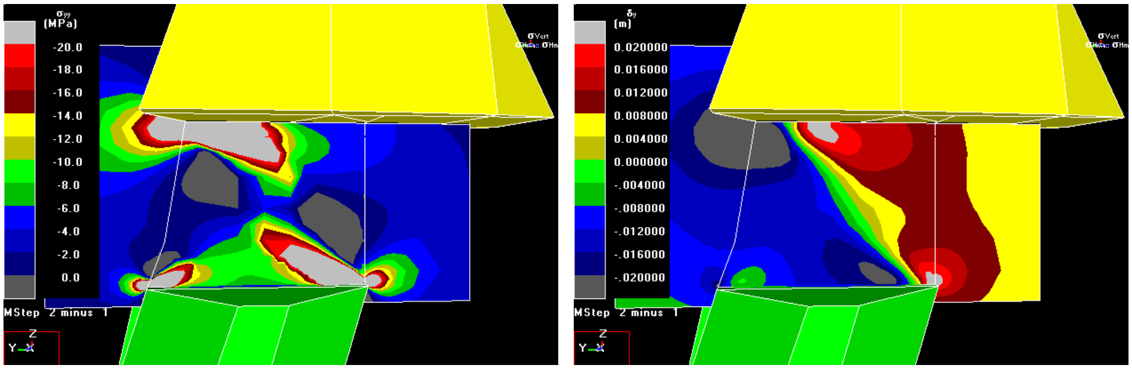

Figure 44: Stress (left) and displacement (right) change due to pillar yielding – fine discretization

In reality there is of course only one solution to this problem amount, with one unique amount of displacement and stress change. For an individual pillar with a set discretization, it is certainly possible to calibrate a simple strain softening model to produce consisent predictions of residual pillar load, magnitude of stress redistribution, strain and displacement. However the flow rule parameters determined would not applicable to other pillars or other models since every situation will have a different discretization profile and hence different shear band characteristics. Unique values for the flow rule parameters can never be determined. Since different models will have a different discretization profile, these will give inconsisent and hence unreliable predictions.

Instead of opting for strain softening response, one could consider specifying a more brittle type failure where the strength suddenly drops to residual values (see Figure 45). Since the stress is no longer a function of the strain, this would eliminate the solution sensitivity to shear band width. It is unknown how one would properly deal with dilations. Perhaps one could simply choose a fixed dilation rate dependant on confining stress.

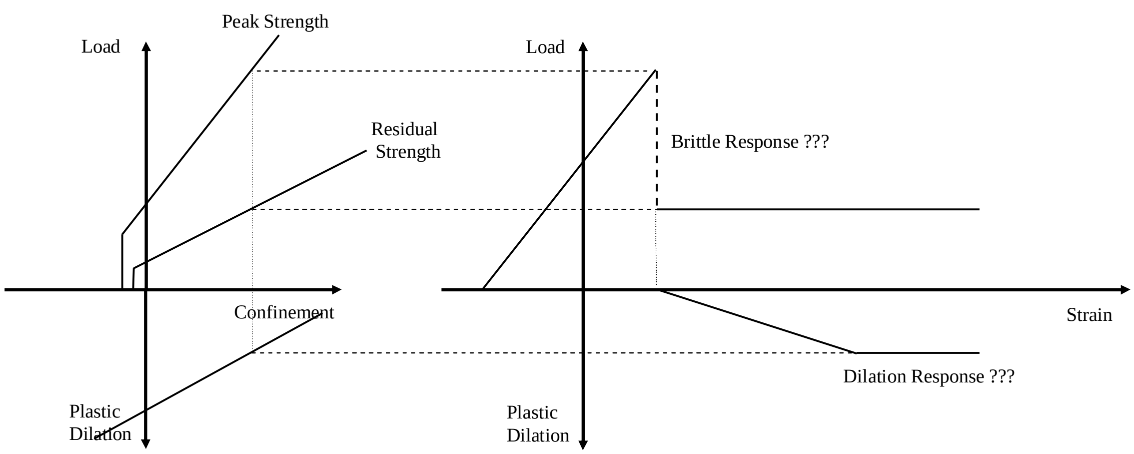

Figure 45: Brittle plastic response

Plasticity modelling theory issue #3 – discetization requirements

In any numerical model, in the area of interest fine discretizations are used to obtain predictions without inaccuracies due to numerical approximation. However as one moves away from the area of interest, quite course discretizations are used in order to reduce the computational burden. Since these later locations are distant from the area of interest, it is thought that the influence of this simplification will be small. This is not necessarily the case in plastic modelling.

Take for example the simple pillar failure model show in the figure below (Figure 46). This model simply consists of a large series of 1x1 parallel drives shown in blue, with intervening 1x1 pillars shown in green. On the left only the central pillar area has been finely discretized, while on the right the entire model has been finely discretized.

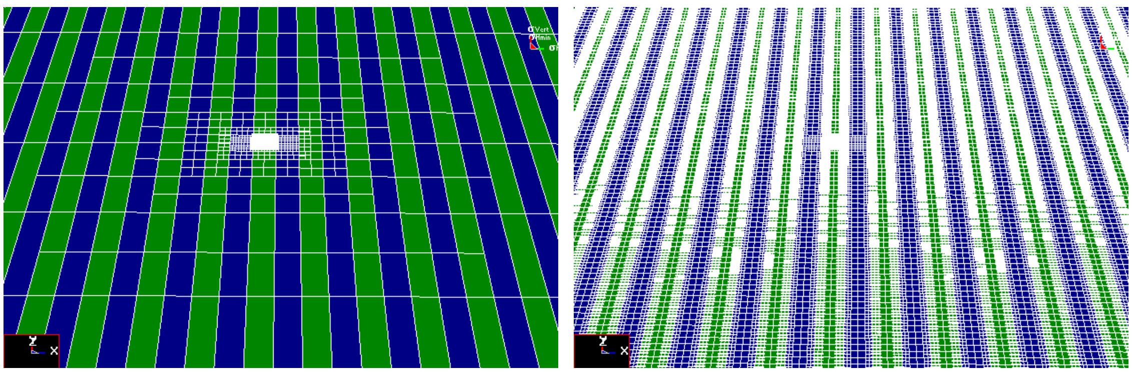

Figure 46: Multiple pillar model with different discretizations

Fine discretization in all pillars would provide the best simulation results since all pillars are modelled with best possible accuracy. However when only the central pillar is finely discretized and coarser discretizations are used in adjacent pillars, the accuracy of the response in the central pillar deteriorates quite markedly. This problem arises because of subtle details of the way in which the adjacent pillars are modelled. With only a very few nodes (plasticity zones or finite elements) in the adjacent pillars, these are not modelled correctly.

To understand this effect, take for example the extreme case where a pillar is modelled with only a single central node. The stress state at this central location is clearly not representative of the pillar. In fact for wide pillars, the central point may not be in a state of failure at all and hence this pillar would not failure and redistribute any load. Where in reality, failure of the side walls could lead to progressive failure of the entire pillar. The difference in these two cases is that these non-failed pillar would redistribute very little stress compared to failed pillars. Hence the stress transfer to the central pillar will not simulated correctly and the predicated behaviour will be in error.

To demonstrate this effect a model has been run with the parameters set the same way as in the example “De-Stressing of a loaded pillar example” above with the pre-mining stresses all set to 30 MPa and UCS=25 MPa and friction angle set to 30°.The plastic dilation rate is set to zero.

Below (Figure 47) are images of the pillar load (major principal stress) in the central pillar. As above, on the left only the central pillar area has been finely discretized, while on the right the entire model has been finely discretized. In spite of the increased loads due to differing amount of stress transfer from adjacent pillars, from a stress point of view the results are nearly identical. This is expected since in plastic modelling, the stress distribution is controlled by the specified strength criterion and flow rule, rather than the loading conditions (these models have been analysed with the Map3D Visco-Plastic analysis program).

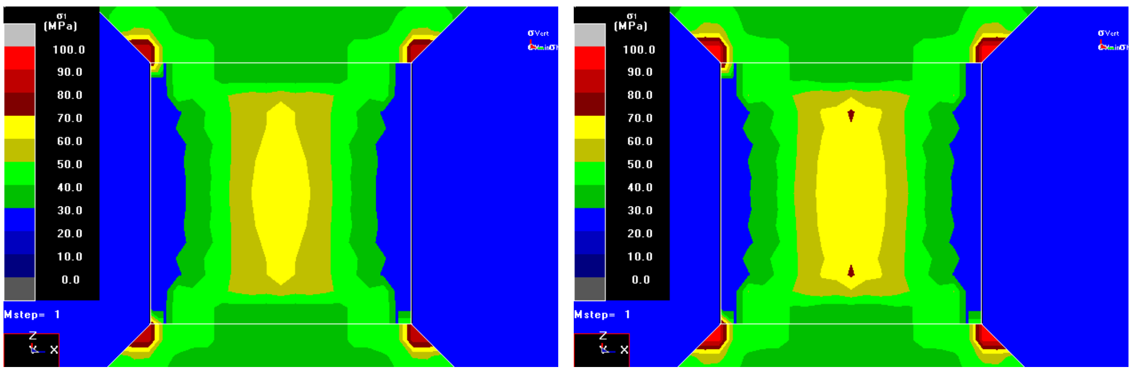

Figure 47: Pillar load (σ_1 - major principal stress) in the central pillar (plastic model)

Now consider the plastic strains (Figure 48) and displacements (Figure 49). It is clearly illustrated that the model that has been finely discretized throughout is predicting much large deformations. The plastic strains and displacements for the uniformly discretized model (on the right) are 40% higher.

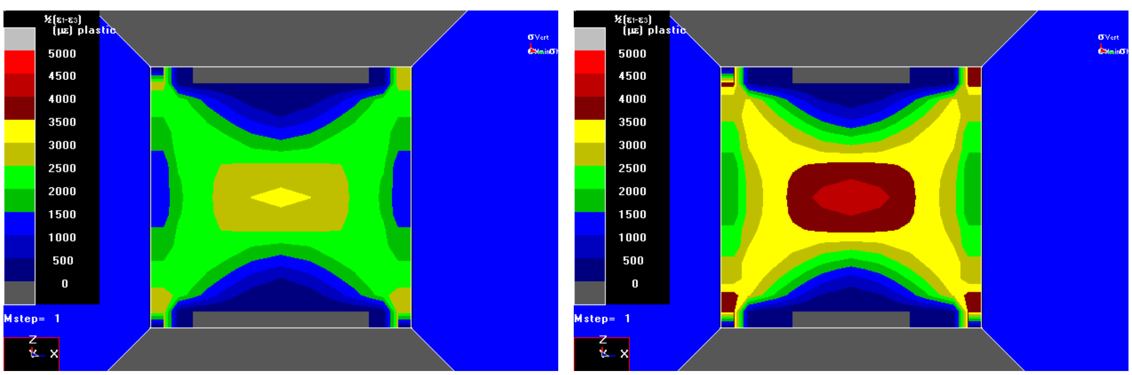

Figure 48: Plastic shear strain ½(ε_1- ε_3) in the central pillar (plastic model)

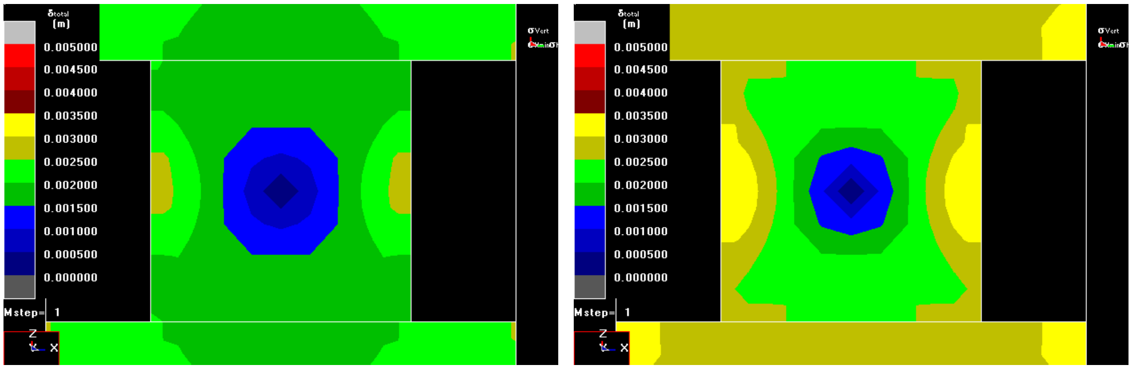

Figure 49: Displacement in the central pillar (plastic model)

The above models have been analysed using an elastic/perfectly plastic flow rule (Figure 38). If these had been analysed with a strain softening model (Figure 39), this effect would be exacerbated.

Note that this effect is not present in elastic models (see Figure 50, left - only central pillar finely discretized, right - all pillars finely discretized).

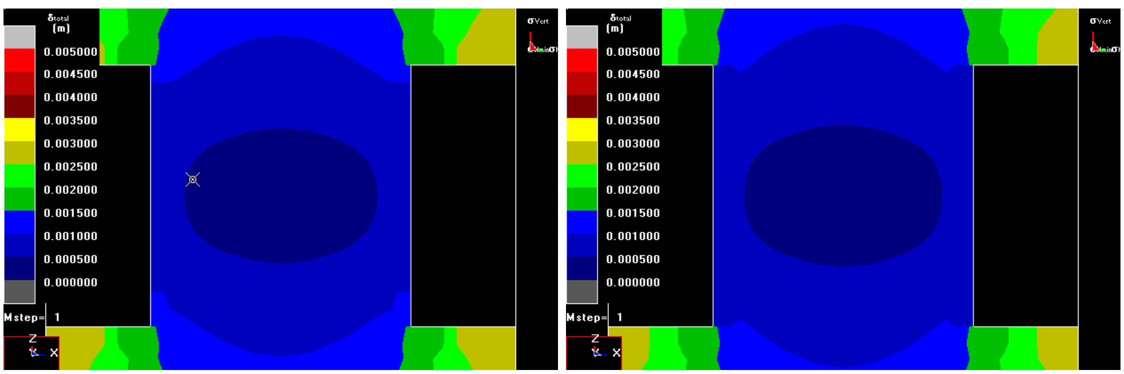

Figure 50: Displacement in the central pillar (elastic model)

The original reason for adopting more complex plastic models was to simulate this exact stress redistribution caused by pillar yielding effect, the justification being that stress redistribution was an important effect that needed to be taken in to account in order to provide predictions representative of in situ response. It is apparent that without fine discretization throughout the plastic model, stress redistribution is simply not modelled correctly.

Plasticity modelling theory issue #4 – stress path

It is important to appreciate that since the mining sequence directly affects the stress path, hence it plays a very important role in the ultimate behaviour of any plasticity model. Unlike in elastic modelling, different mining sequences can result in entirely different end results. To demonstrate this effect consider a simple model where mining makes a close pass to a pre-existing drift or shaft as shown in Figure 51.

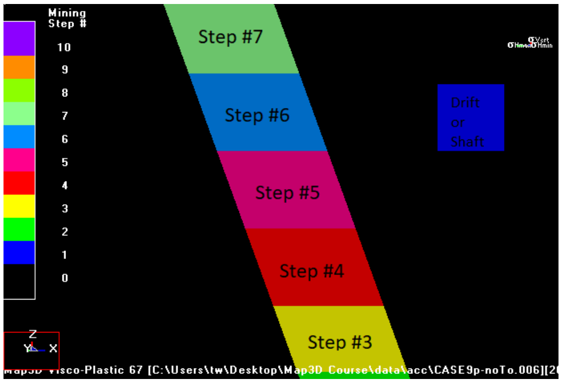

Figure 51: Mining near a drift or shaft

If the excavation is advanced by combining all of steps #1 through #6 as a single excavation step, then the resulting plastic strain (damage) is limited to a few corners as shown in Figure 52.

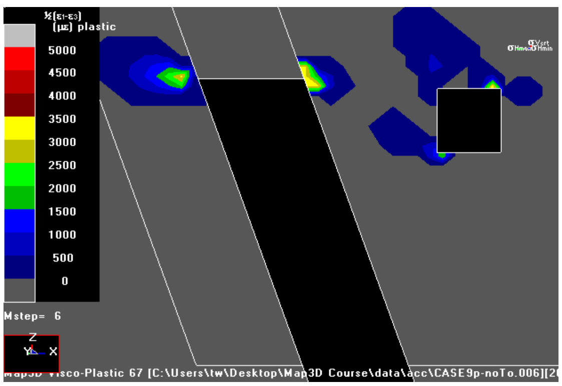

Figure 52: Combine all of steps #1-#6 as a single excavation

Whereas if the true mining sequence is carried out in detail by excavating all steps individually, an entirely different result is found as shown in Figure 53.

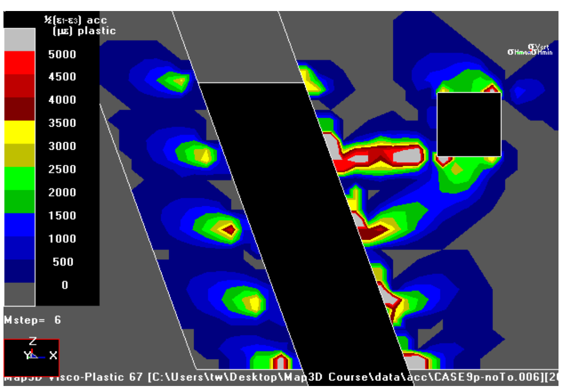

Figure 53: Mine steps #1-#6 individually

Note in particular the shear band that has formed between the mining and the drive at step #5. This suggests considerable damage to the drive and probably instability in the hanginging wall of the mining that is only evident in the multi-step model. This behaviour is a result of different stress paths being followed and clearly demonstrates how important it is to model the actual mining sequence in detail.

By their vary nature, non-linear equations can provide non-unique results as illustrated in the example above. These problems are not limited to mining sequence alone. Owing to the non-linear nature of plasticity equations, solution requires an iterative process. This process is inherently unstable and must be controlled. There are many ways of going about this and various plastic modelling programs all approach this in a different manner. What are referred to as either implicit or explicit solution schemes are used to solve this non-linear set of simultaneous equations.

One method of going about this is to gradually reduce the excavation boundary stresses from the pre-mining stress level to zero. By doing this in a series of small steps, the increments of excess elastic stress will be small, and hence the plastic deformation can be controlled well enough to follow the flow rule.

Another method is to suddenly dropped the excavation boundary stresses to zero. This can be modelled as a large excess elastic stress that can be broken into smaller increments either by gradual reduction in the strength from an artificially high value to the actual specified strength, or by viscous time stepping where the excess stress is relieved in a series of controlled plastic increments proportional to the magnitude of the excess stress. Some programs use damped inertial time stepping to control strain rates in a similar manner.

Because of the above issues, one must realize that there is not necessarily one unique solution to any set of non-linear equations. Each of the above alternatives can result in a different end result. This is because for each solution method, the stress path at any location will be different, and hence different amounts of plastic strain will occur at different locations at different times. The resulting stresses, strains and displacements will be distributed differently. In comparison studies of various plastic modelling programs it is not uncommon to find varying predictions from different analysis programs for reasons such as this.

Plasticity modelling theory – concluding remarks

The motivation for going beyond elastic modelling and introducing plastic modelling was to get a more accurate estimate of the stress redistribution and displacements. Although it is true that plasticity theory offers the most realistic method of achieving this, from a practical point of view it would appear that this approach is an excessively complicated, unreliable and inefficient method to accomplish something as simple as stress redistribution. Although complex simulations offer the potential to provide the best simluation of rock response, keep in mind that these will come with a lack of demonstratable reliability. Owing to the large number of input parameters, thorough calibration of such models are rarely if ever carried out. This approach may not be the best way of solving this problem.