MAP3D VERIFICATION EXAMPLE 1

Cylindrical Hole in an Infinite Elastic Medium

1.1 Description

This verification example involves calculating the stresses and displacements for a cylindrical hole in an infinite isotropic elastic medium subjected to a hydrostatic stressfield.

The model input parameters are based on an example contained in the Flac2D Verification manual[1].

The MAP3D model geometry is shown in Figure 1 with solution grids located at the mid-length of the cylindrical hole.

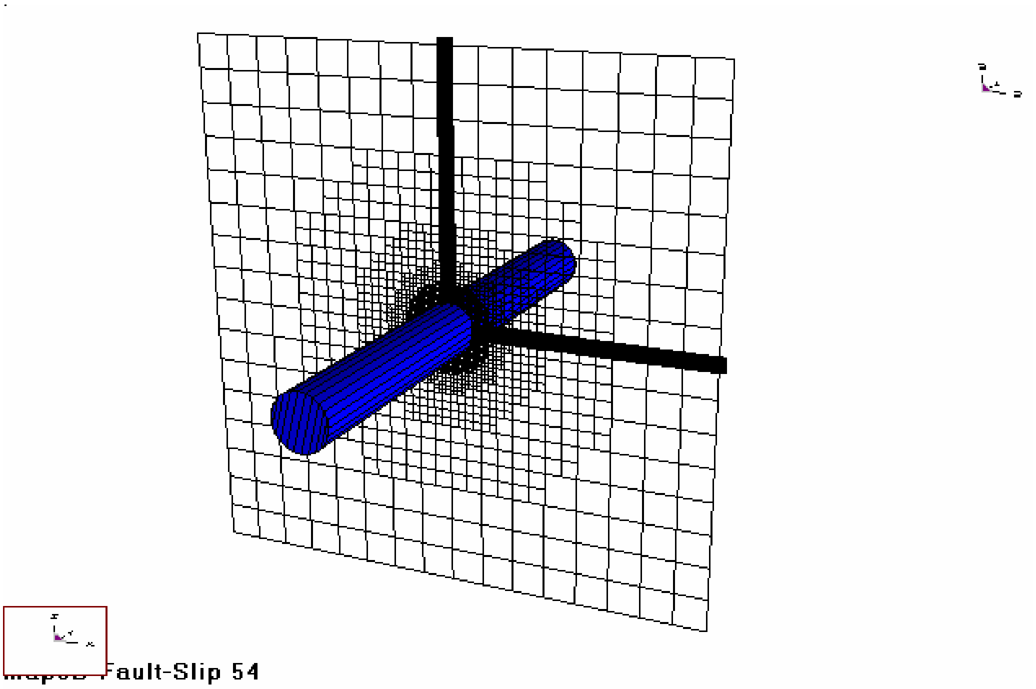


Figure 1: The MAP3D Cylindrical Hole model.

The radius of the hole is set at one metre and is relatively small compared to the overall length of the cylinder. This provides solutions at the mid side of the 3D cylinder that are equivalent to a 2D plane strain condition and can be compared to appropriate analytical solutions (see later).

[1] Itasca Consulting Group, INC (2002), Fast Lagrangian Analysis of Continua (Version 4), Verification Manual - Cylindrical Hole in an Infinite Elastic Medium.

The 3D cylindrical hole model is defined by fictitious force elements. There is no need to define an external boundary in MAP3D as required by both the Flac2D and Phase2 programs.

This is one of the many benefits of the MAP3D boundary element formulation. Note also that the symmetry option in MAP3D has not been used in this example.

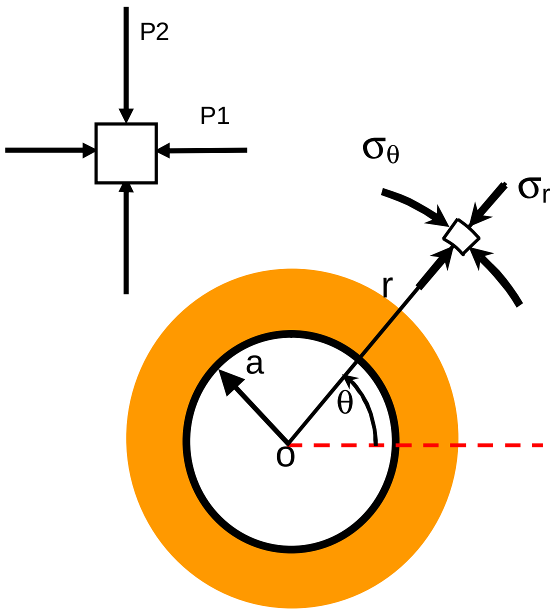
The host material is defined as isotropic with a Young's modulus of 6,778 MPa and a Poisson's ratio of 0.21. These values are derived from the original Flac2D example material values of a shear modulus (G) of 2,800 MPa and a bulk modulus (K) of 3,900 MPa.

The example uses a constant hydrostatic compressive in-situ stress field of 30 MPa. ($P_1 = P_2$). It is also assumed that there is no pressure inside the hole.

1.2 Closed Form Solutions

Cylindrical Hole in an Infinite Elastic Material

The classical Kirsch solution can be used to find the radial and tangential displacement fields and stress distributions, for a cylindrical hole in an infinite isotropic elastic medium under plane strain conditions (e.g. see Jaeger and Cook, 1976).



The stresses σ_r, σ_θ and $\tau_{r\theta}$, for a point at polar coordinate (r,θ) near the cylindrical opening of radius 'a', are given by the following equations:

$$\sigma_r = \frac{p1+p2}{2}\left(1-\frac{a^2}{r^2}\right)+\frac{p1-p2}{2}\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{p1+p2}{2}\left(1+\frac{a^2}{r^2}\right)-\frac{p1-p2}{2}\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\tau_{r\theta} = -\frac{p1-p2}{2}\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta$$

The radial outward and tangential displacements, υ_r and υ_θ respectively, can also be determined assuming plane strain conditions. Here *G* is the shear modulus, and ν is the Poisson's ratio

$$u_r = \frac{p1+p2}{4G}\frac{a^2}{r}+\frac{p1-p2}{4G}\frac{a^2}{r}\left(4(1-\nu)-\frac{a^2}{r^2}\right)\cos 2\theta$$

$$u_\theta = -\frac{p1-p2}{4G}\frac{a^2}{r}\left(2(1-2\nu)+\frac{a^2}{r^2}\right)\sin 2\theta$$

1.3 Results and Discussion

The tangential or hoop stresses σ_θ (s1) and the radial σ_r (s3) stresses calculated by MAP3D are compared to the analytical solution along a line in the radial direction as shown in Figure 2.

The radial (outward) displacement u_r calculated by MAP3D is compared to the analytical solution as shown in Figure 3.

A radial distance of 1m is at the cylinder's surface.

The MAP3D results are in very close agreement along a line through the model with the equivalent analytical solutions.

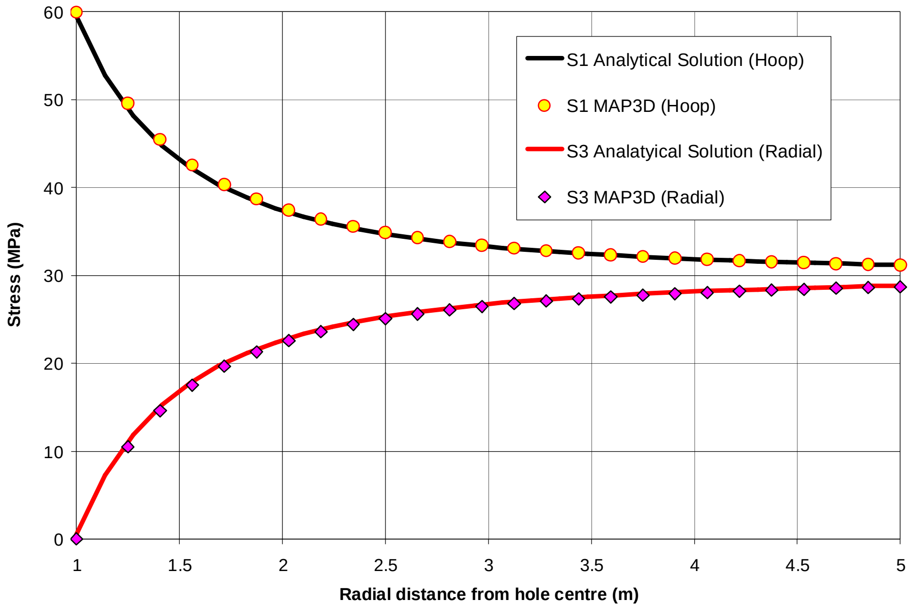


Figure 2: Comparison of radial and tangential stresses, σ_r (S_3) and $\sigma_{\theta.}$(S_1) plotted versus radius.

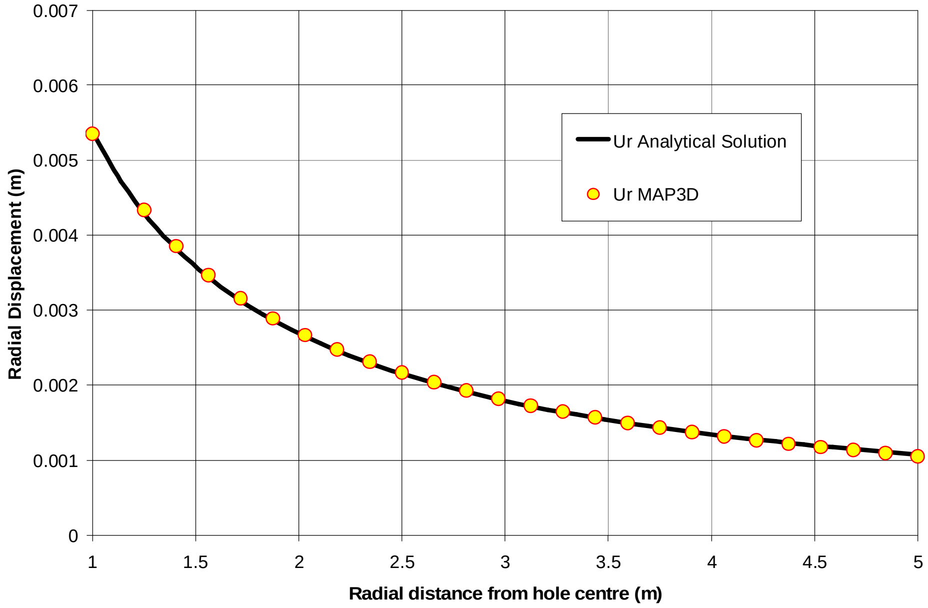


Figure 3: Comparison of radial displacement u_r from the centre of the hole.

Contours of the principal stresses σ1 (tangential or hoop stresses σ_θ) and σ3 (radial stress σ_r) on the solution grid located at mid-length of the cylinder are presented in Figures 4 through 7.

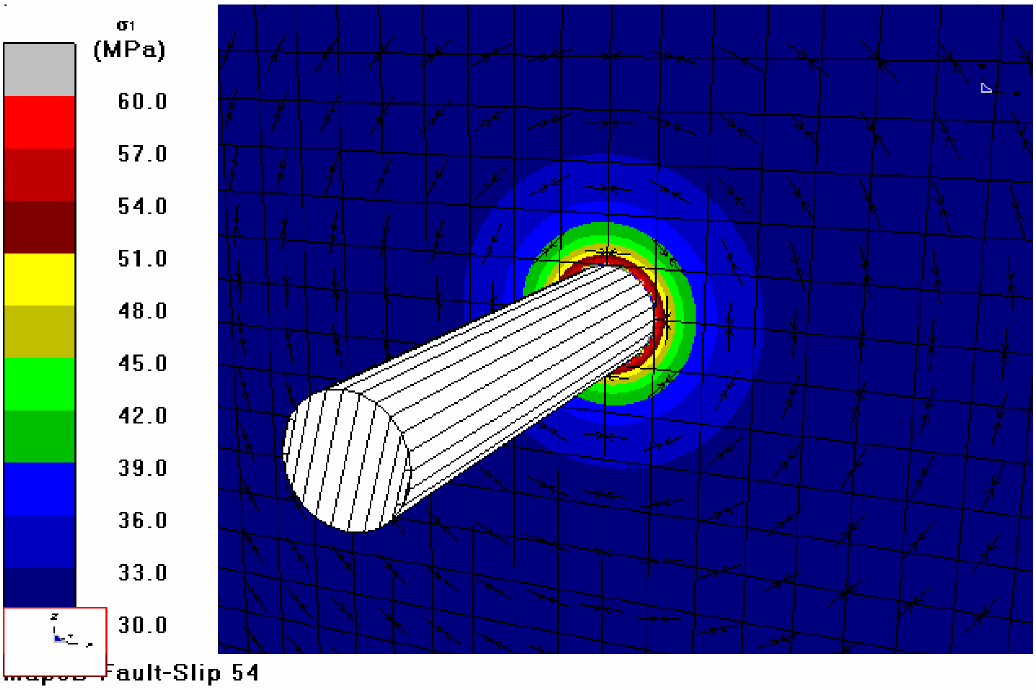


Figure 4: Contours of σ1 (hoop stresses) on a grid plane at the mid-length of the cylindrical hole.

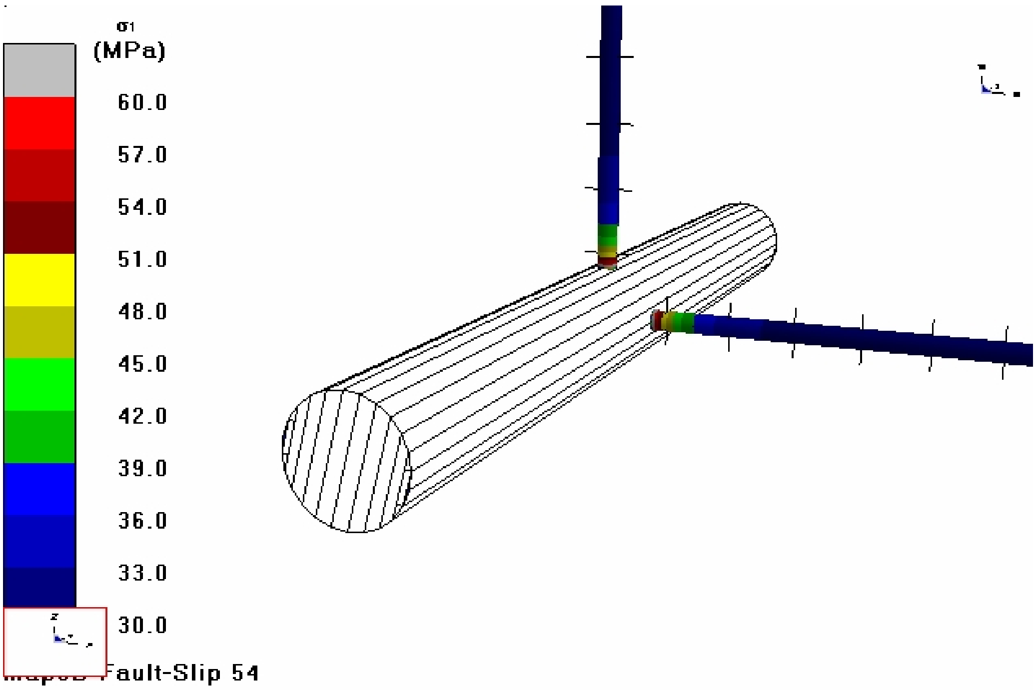


Figure 5: σ1 (hoop stresses) plotted on line grids at the mid-length of the cylindrical hole.

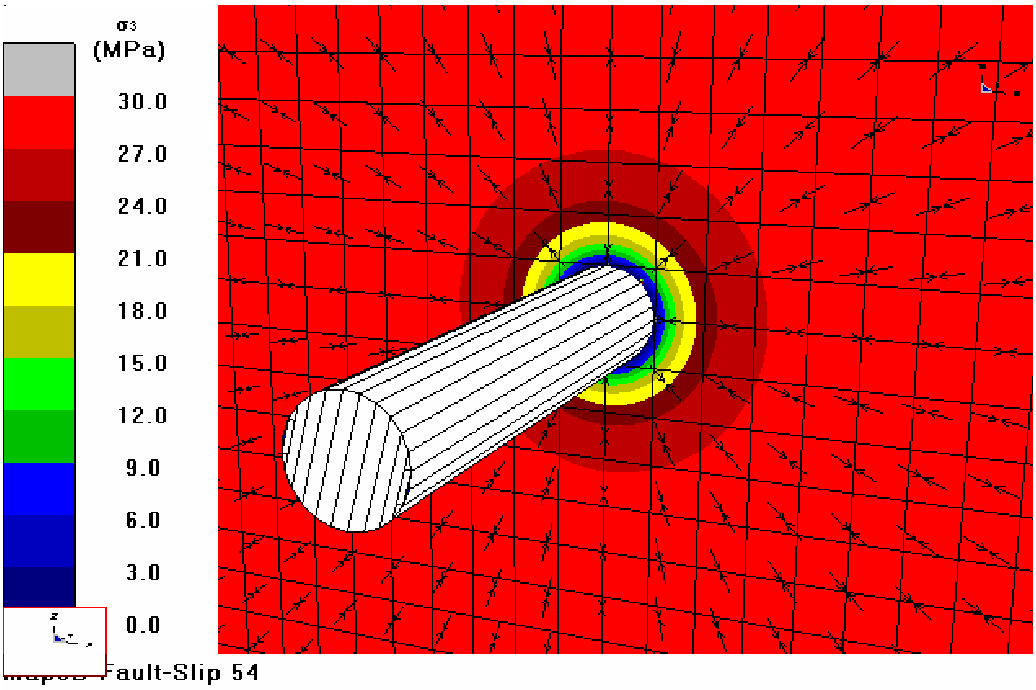


Figure 6: Contours of σ3 (radial stresses) on a grid plane at the mid-length of the cylindrical hole.

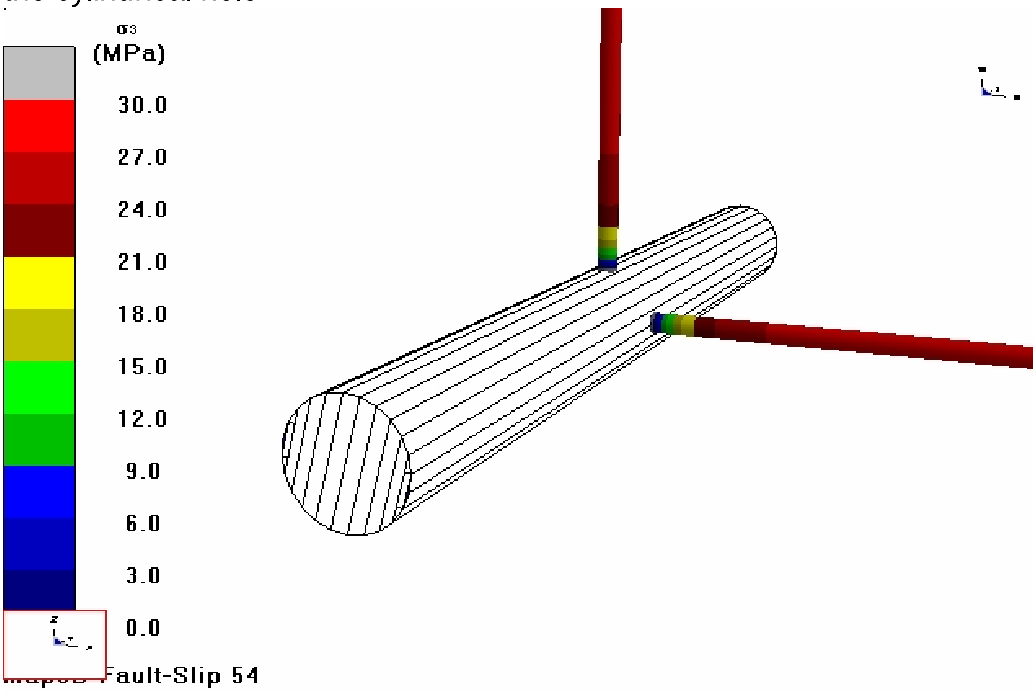
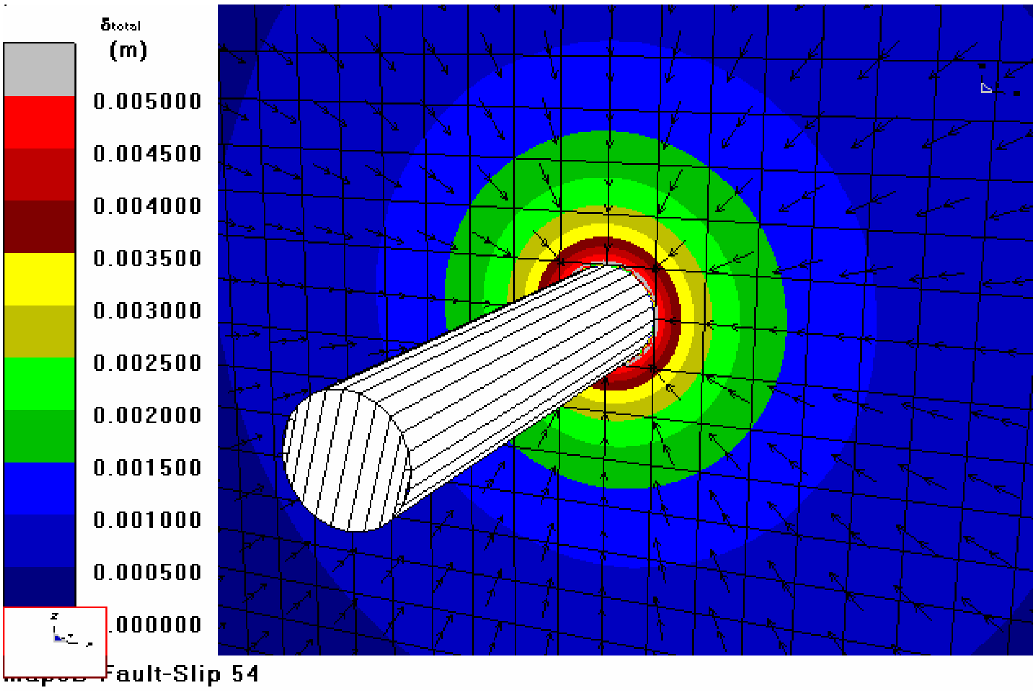
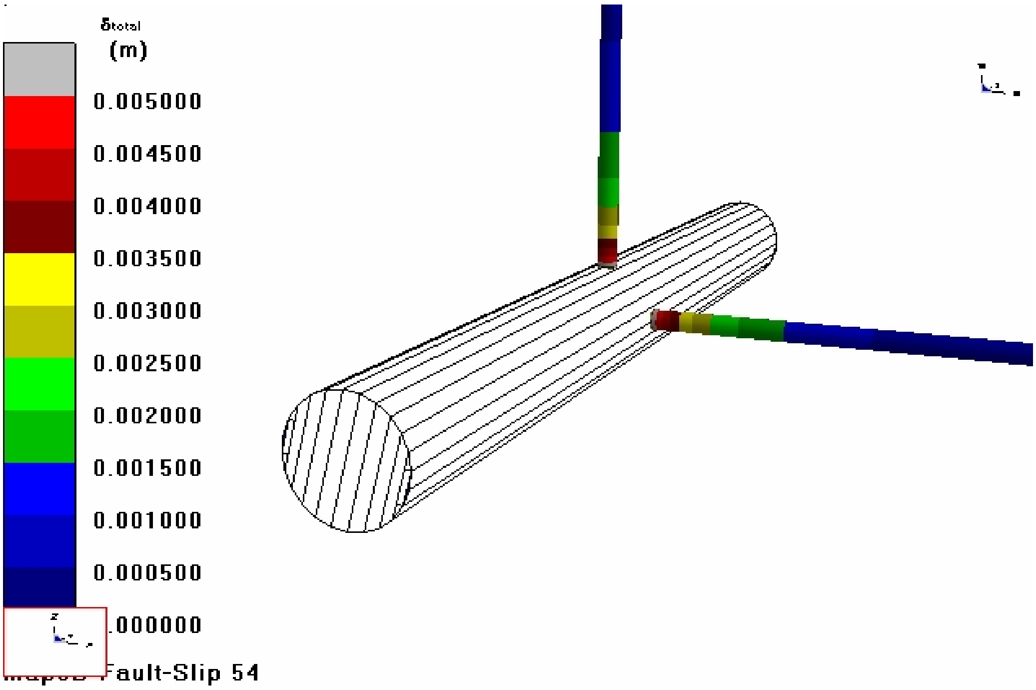


Figure 7: σ3 (radial stresses) plotted on line grids at the mid-length of the cylindrical hole.

Contours of the radial (outward direction) displacement distribution are illustrated in the following Figure 8 and Figure 9.



Figures 8 Radial (outward) displacement plotted on a grid plane at the mid-length of the cylindrical hole.



Figures 9 Radial (outward) displacement plotted on line grids at the mid-length of the cylindrical hole.

1.5 References

Jaeger, J.C. and N.G.W. Cook. (1976) Fundamentals of Rock Mechanics, 3rd Ed. London

Kirsch G. Die Theorie der Elastizitat und die Bedurfnisse der Festigkeitslehre, Zantralblatt Verlin Deutscher Ingenieure, 42 (1898) 797-807.

2.6 MAP3D Data Files

The accompanying **Cylindrical Hole Tutorial** and input data file can be used to replicate the results.

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