MAP3D VERIFICATION EXAMPLE 5

Cylindrical Hole in a Mohr-Coulomb Medium

1.1 Description

This verification example involves calculating the stresses and displacements for a cylindrical hole in a Mohr-Coulomb medium subjected to a hydrostatic stressfield.

The model input parameters are based on an example contained in the Flac2D Verification manual[1]. This example also appears in the Rocscience Phase2 Verification manual.

The MAP3D model geometry is shown in Figure 1 with a solution grid located at the mid-length of the cylindrical hole (blue zone) surrounded by a Mohr-Coulomb 'plastic material region' (green zone). It is within this zone that yielding can take place wherever stresses exceed the designated strength.

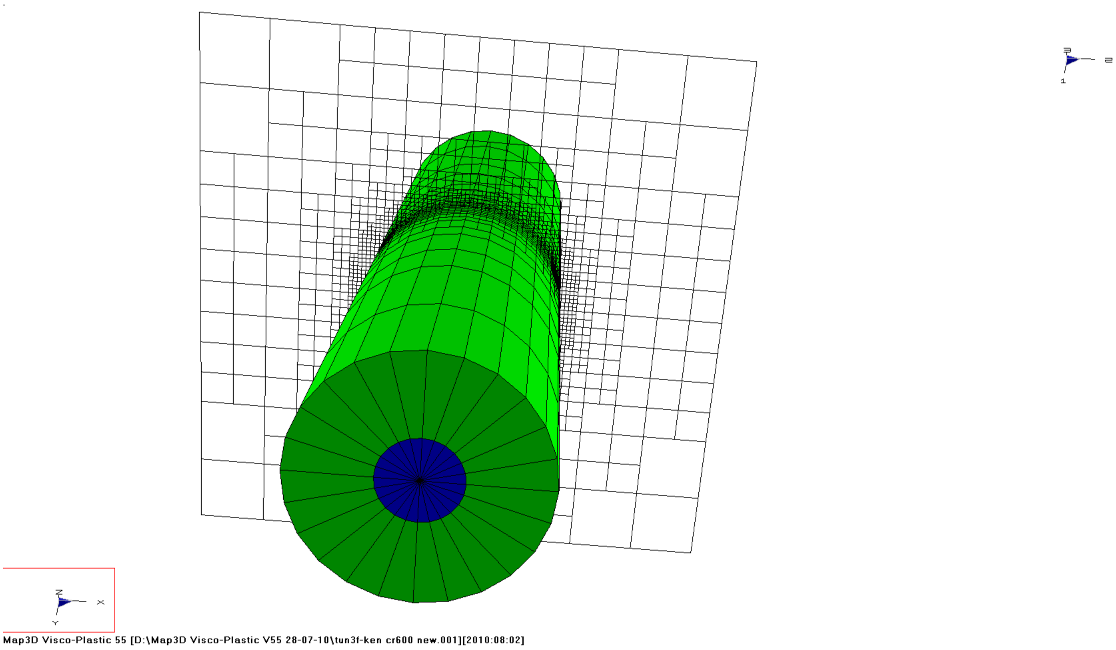


Figure 1: The MAP3D Cylindrical Hole model.

The radius of the hole is set at one metre and is relatively small compared to the overall length of the cylinder. This provides solutions at the mid side of the 3D cylinder that are equivalent to a 2D plane strain condition and can be compared to appropriate analytical solutions (see later).

[1] Itasca Consulting Group, INC (2002), Fast Lagrangian Analysis of Continua (Version 4), Verification Manual - Cylindrical Hole in an Infinite Mohr-Coulomb medium.

The 3D cylindrical hole model is defined by fictitious force elements. There is no need to define an external boundary in MAP3D as required by both the Flac2D and Phase2 programs.

This is one of the many benefits of the MAP3D boundary element formulation. Note also that the symmetry option in MAP3D has not been used in this example.

The host material is defined as isotropic with a Young's modulus of 6,778 MPa and a Poisson's ratio of 0.21. These values are derived from the original Flac2D example material values of a shear modulus (G) of 2,800 MPa and a bulk modulus (K) of 3,900 MPa.

The plastic material zone is also constructed from fictitious force element blocks as shown in Figure 2 (green blocks). The figure has some faces hidden to show the internal nodes within the plastic zone concentrated near the solution grid.

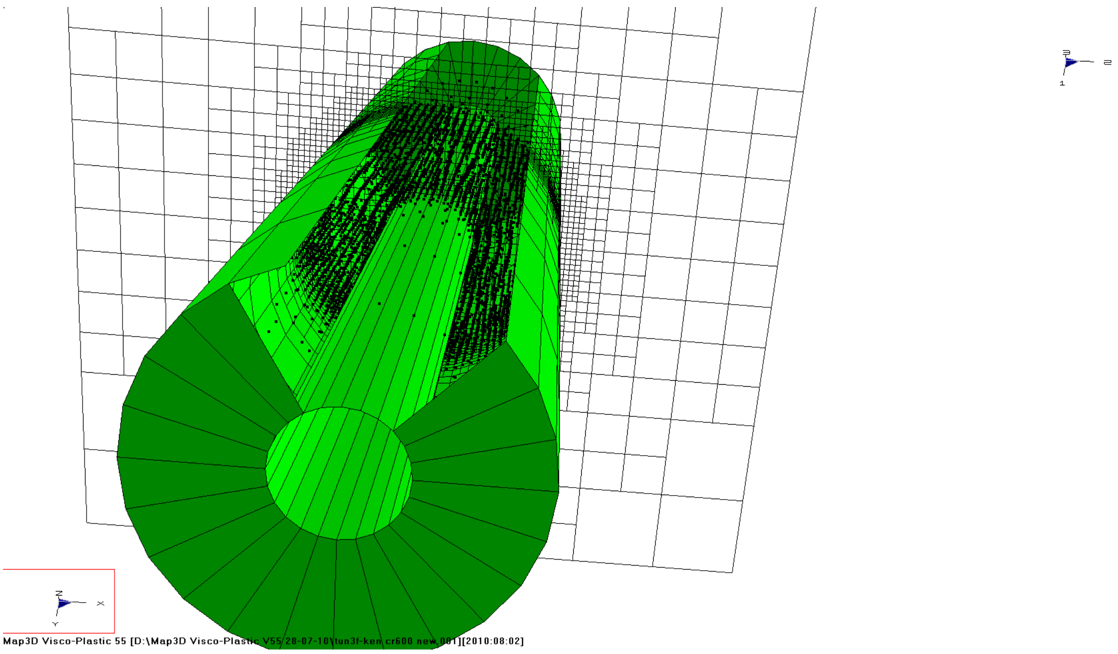


Figure 2: The plastic material zone surrounding the cylindrical hole.

The outer radius is set larger than the expected yield zone (as can be determined if necessary by a MAP3D-FS analysis). In this case it was set at 3 metres.

The plastic material zone is assumed to be linearly elastic, perfectly plastic, with a failure surface defined by the Mohr-Coulomb criterion with cohesion of 3.45 MPa and a friction angle of 30° for a UCS of 11.95 MPa.

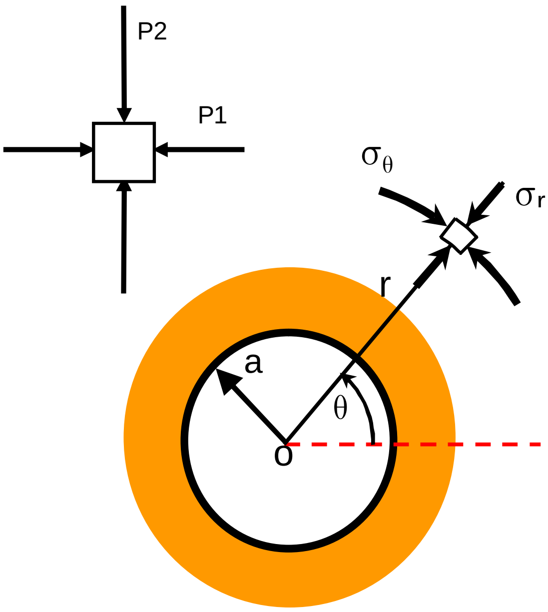
The Viscous Modulus (Gs) is set to 10% of Young's modulus. This produces a loading path that provides a match to the analytical solution. Larger values of Gs simply take longer to arrive at the same solution.

This analysis is for the non-associated (friction angle independent) flow rule case with a dilation angle of 0°.

The example uses a constant hydrostatic compressive in-situ stress field of 30 MPa. ($P_1 = P_2$). It is also assumed that there is no pressure inside the hole.

1.2 Closed Form Solutions

The analytic solution for this problem is based on a theoretical model by Salençon (1969).



Where:

- a = cylindrical hole radius
- c = cohesion
- ϕ = friction angle
- $K_p = \left(\frac{1+\sin\phi}{1-\sin\phi}\right)$
- $q = 2c\tan(45+\phi/2)$
- P_0 = initial in-situ stress
- P_1 = internal pressure (= 0 MPa for the excavation)

The yield zone radius R_0:

$$R_0 = a\left(\frac{2}{K_p+1}\frac{P_0+\frac{q}{K_p-1}}{P_1+\frac{q}{K_p-1}}\right)^{1/(K_p-1)}$$

The stresses σ_r, σ_θ for a point at polar coordinate (r,θ) near the cylindrical opening of radius 'a', are given by the following equations:

The radial stress at the elastic/plastic interface is:

$$\sigma_{re} = \frac{1}{K_p + 1}(2P_0 - q)$$

The radial stresses and displacements in the elastic zone are:

$$\sigma_r = P_0 - (P_0 - \sigma_{re})\left(\frac{R_0}{r}\right)^2$$

$$\sigma_\theta = P_0 + (P_0 - \sigma_{re})\left(\frac{R_0}{r}\right)^2$$

$$\upsilon_r = \frac{R_0^{\ 2}}{2G}\left(P_0 - \frac{2P_0 - q}{K_p + 1}\right)\frac{1}{r}$$

The radial stresses and displacements for the plastic zone are:

$$\sigma_r = -\frac{q}{K_p - 1} + \left(P_i + \frac{q}{K_p - 1}\right)\left(\frac{r}{a}\right)^{(K_p - 1)}$$

$$\sigma_\theta = -\frac{q}{K_p - 1} + K_p\left(P_i + \frac{q}{K_p - 1}\right)\left(\frac{r}{a}\right)^{(K_p - 1)}$$

The radial outward and displacement, υ_r can be determined using the following equation. Here *G* is the shear modulus, and ν is the Poisson's ratio.

The dilation angle is ψ and $K_{ps} = \frac{1 + \sin\psi}{1 - \sin\psi}$

$$\upsilon_r = \frac{r}{2G}\left[\begin{array}{l}(2\nu - 1)\left(P_0 + -\frac{q}{K_p - 1}\right) + \frac{(1-\nu)(K_p^{\ 2} - 1)}{K_p + K_{ps}}\left(P_i + \frac{q}{K_p - 1}\right)\left(\frac{R_0}{a}\right)^{(K_p - 1)}\left(\frac{R_0}{r}\right)^{(K_{ps} + 1)} \\ + \left(\frac{(1-\nu)(K_p K_{ps} + 1)}{K_p + K_{ps}} - \nu\right)\left(P_i + \frac{q}{K_p - 1}\right)\left(\frac{r}{a}\right)^{(K_p - 1)}\end{array}\right]$$

1.3 Results and Discussion

The tangential or hoop stresses σ_θ (s1) and the radial σ_r (s3) stresses calculated by MAP3D are compared to the analytical solution along a line in the radial direction as shown in Figure 3.

The radial (outward) displacement u_r calculated by MAP3D is compared to the analytical solution as shown in Figure 4.

A radial distance of 1m is at the cylinder's surface.

The MAP3D results are in very close agreement along a line through the model compared with the equivalent analytical solutions.

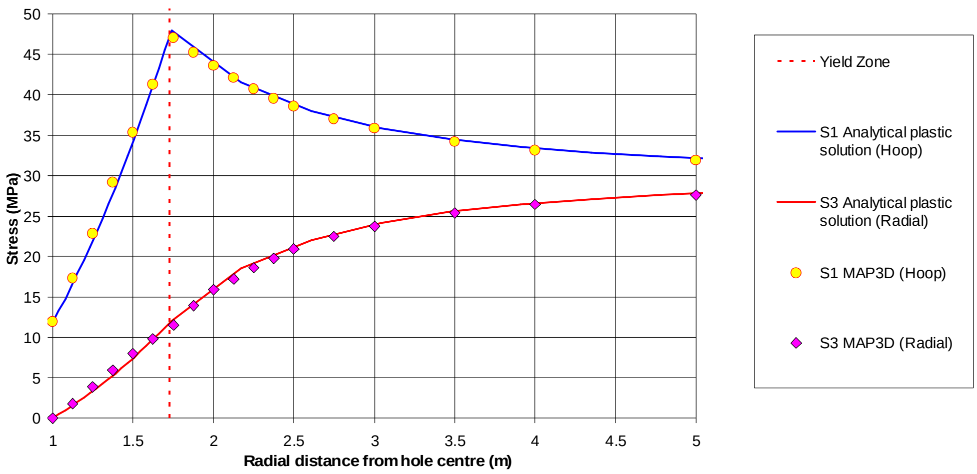


Figure 3: Comparison of radial and tangential stresses, σ_r (S_3) and σ_θ.(S_1) plotted versus radius.

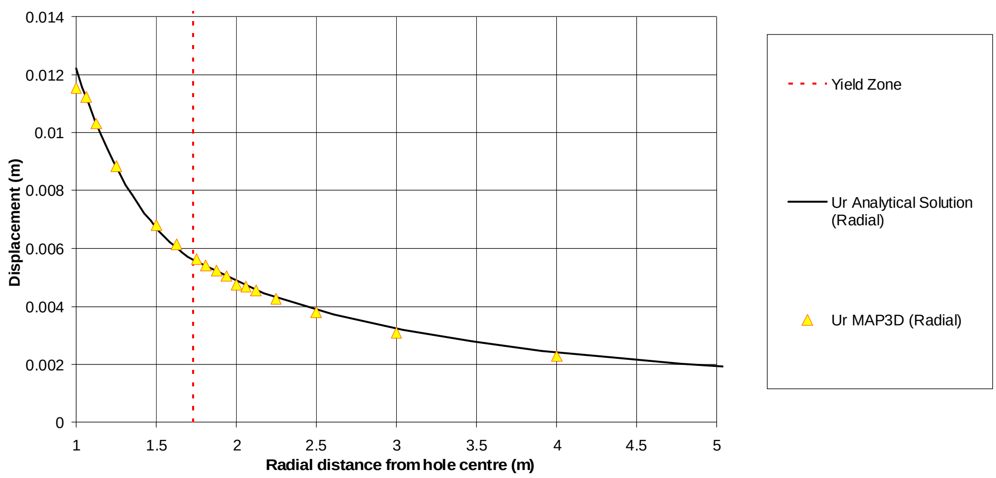


Figure 4: Comparison of radial displacement u_r from the centre of the hole.

Contours of the principal stresses σ1 (tangential or hoop stresses σ_θ) and σ3 (radial stress σ_r) and u_r radial displacement (outward direction) distribution on the solution grid located mid-length of the cylinder are shown in Figures 5, 6 and 7.

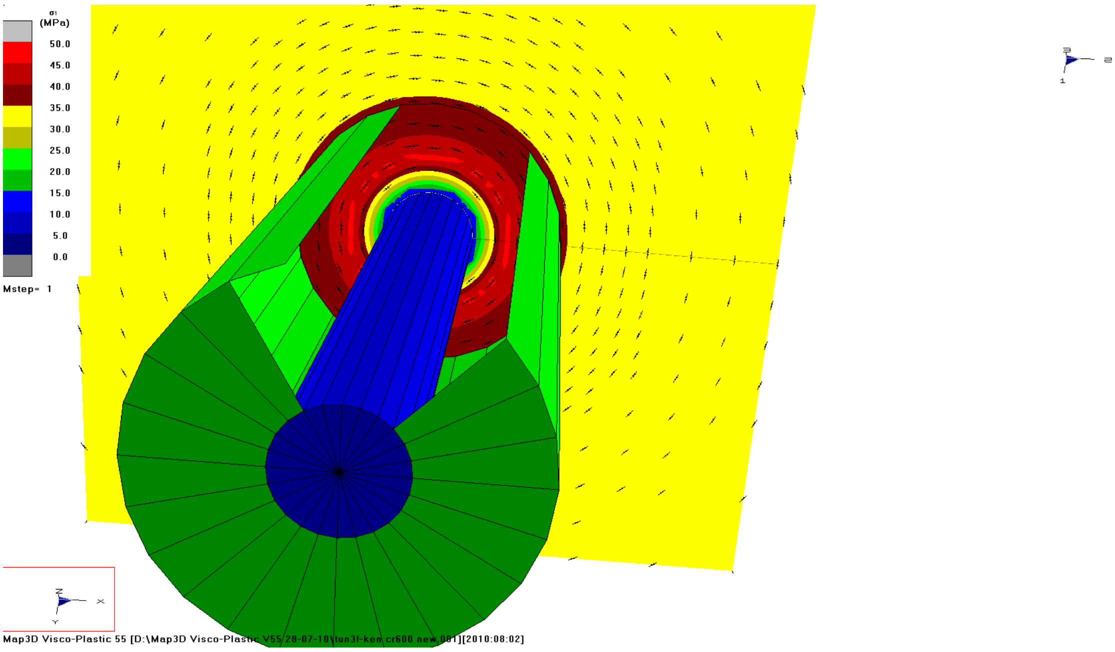


Figure 5: Contours of σ1 (hoop stresses) on a grid plane at the mid-length of the cylindrical hole. The plot includes the plastic zone geometry.

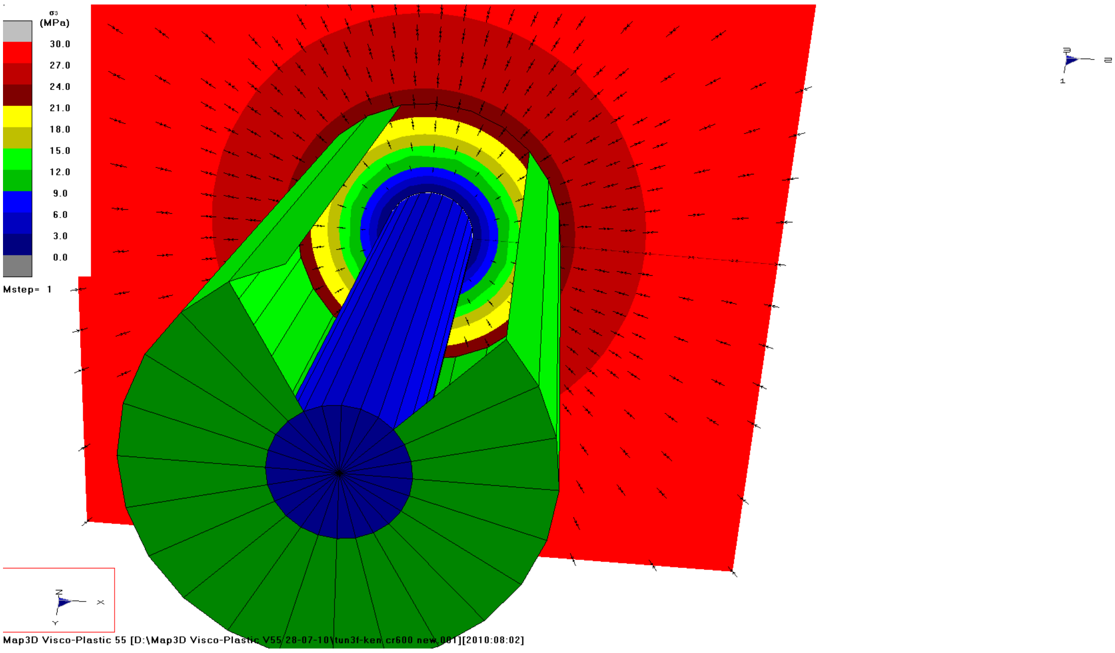


Figure 6: Contours of σ3 (radial stresses) on a grid plane at the mid-length of the cylindrical hole.

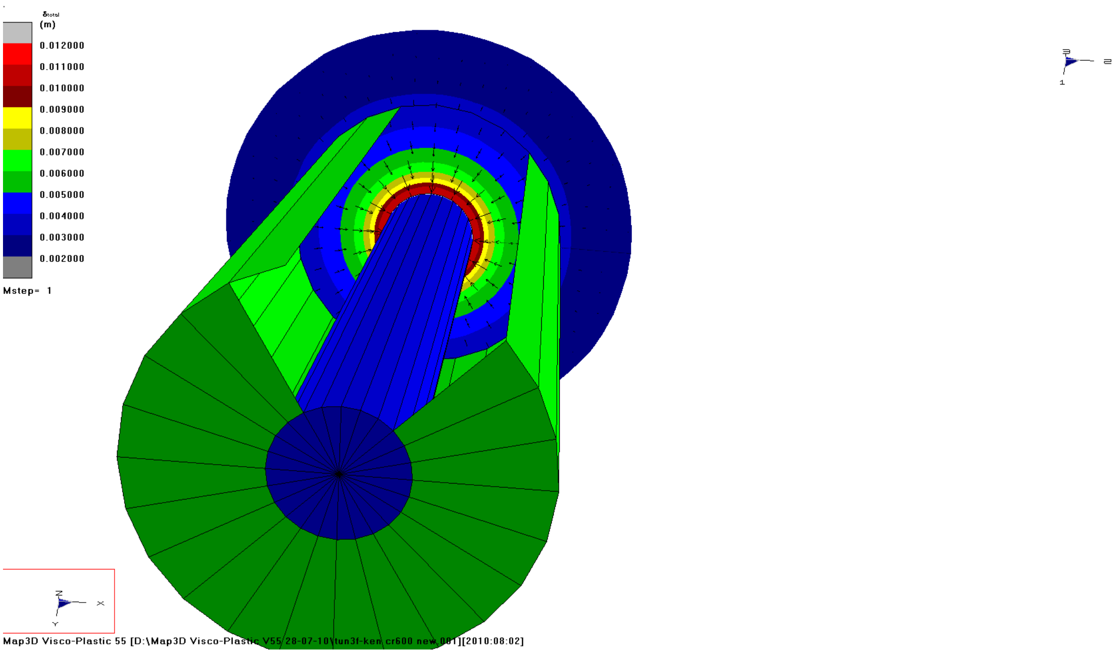


Figure 7 Contours of u_r radial displacements (outward) plotted on a grid plane at the mid-length of the cylindrical hole.

1.4 MAP3D Data Files

The accompanying **Cylindrical Hole Tutorial** and input data file can be used to replicate the results. The input data file for the Cylindrical Hole in an Mohr-Coulomb Medium is: tun3f-ken cr300.inp (Non-Associated flow)

1.5 References

Salencon, J. (1969), Contraction Quasi-Statique D'une Cavite a Symetrie Spherique Ou Cylindrique Dans Un Milieu Elasto-Plastique, Annales Des Ports Et Chaussees, Vol. 4, pp. 231-236.

Itasca Consulting Group, INC (1993), Cylindrical Hole in an Infinite Mohr-Coulomb Medium, Fast Lagrangian Analysis of Continua (Version 3.2), Verification Manual.

Rocscience Phase2 Verification Manual