## Strip Footing on Surface of Mohr-Coulomb Material

## Problem description

The accurate prediction of collapse loads by a numerical model under steady plastic flow conditions can be difficult (Sloan and Randolph 1982).
Potential numerical problems include overestimating the failure load ("locking" phenomenon) and also not reaching a failure load in some cases.

A classic problem involving steady plastic flow is the determination of the bearing capacity of a flexible strip footing sitting on an elastoplastic medium. The bearing capacity is obtained when steady plastic flow has developed beneath the footing and provides a good test of the program to model this condition. A typical application is assessing pillar punching.

The classic closed form solution for the collapse load ' $q$ ' derived by Prandtl for this problem is used for comparison to the MAP3D solution (see below).

The strip footing with a width of 6 m is located on an initially unstressed elastoplastic MohrCoulomb material with the following properties:

Young's modulus $\mathrm{E}=257.143 \mathrm{MPa}$.
Poisson's ratio $=0.285714$
Cohesion (c) $=0.1 \mathrm{MPa}$
Friction angle $(\varphi)=0$
Dilatancy angle $(\Psi)=0^{\circ}$ (incompressible plastic flow).

The collapse load ' $q$ ' from 'Prandtl's Wedge' solution can be found in Terzaghi and Peck (1967):

$$
\begin{aligned}
& q=(2+\pi) \mathrm{c}, \text { where } \mathrm{c} \text { is the cohesion of the material. } \\
& \cong 514 \mathrm{c} \text { or } 0.514 \mathrm{MPa} \text { using a cohesion of } 0.1 \mathrm{MPa} .
\end{aligned}
$$

The plastic flow region under the flexible footing is shown in Figure 1.


Figure 1 Prandtl's wedge problem - failure mechanism under strip loading on a frictionless medium.

## Model

The MAP3D-VP model geometry used for this problem is shown in Figure 2.
The vertical loading is applied in a series of steps to the 6 m wide strip footing shown as green.

The Mohr-Coulomb elastoplastic material 'hex-zone' is constructed from fictitious force element blocks and is shown in tan. The zone extends 10 m 's beneath the ground surface shown as blue.

The vertical solution grid used in the model beneath the footing is also shown.


Figure 2 Model used for the MAP3D-VP analysis

## Results and Discussion

The total displacements beneath the centre of the flexible footing have been plotted against the magnitude of the applied vertical loading in Figure 3.

The plot includes the analytical solution for the collapse load for a footing on a purely cohesive medium as given by $(2+\pi) \mathrm{c} \approx 5.14 \mathrm{c}$ (or 0.514 MPa in this case).

This plot shows the history of the bearing capacity of the footing through to collapse.

Figure 4 combines the results of number analyses with alternative loading paths to demonstrate the ability of MAP3D-VP to match the analytical solution.


Figure 3 Pressure vs. displacement response for a flexible strip footing on a Mohr-Coulomb material.


Figure 4 Pressure vs. displacement response for a flexible strip footing on a Mohr-Coulomb material for a number of analyses with different load paths.

Contours of vertical displacement and plastic strain beneath the footing on the vertical solution grid beneath the footing are shown on Figure 5.


Figure 5 Contours of vertical displacement and plastic strain beneath the footing.
The MAP3D-VP results are in very close agreement with the analytical solution.

## References

Sloan, S. W., and Randolph M. F.. Numerical Prediction of Collapse Loads Using Finite Element Methods, Int. J. Num. \& Analy. Methods in Geomech., 6, 47-76, 1982.

Terzaghi, K., and Peck R. B.. Soil Mechanics in Engineering Practice, 2nd Ed. New York: John Wiley and Sons, 1967

